CSAP/PREC

COLLEGE STUDENT ACHIEVEMENT PROJECT

Bridging the Mathematics Gap through Learning Outcomes

Final Report

For the

Ontario Ministry of Training, Colleges and Universities

Patricia Byers, PhD

Project Lead

Preface

This final report contains an overview of activity and work completed by the Learning Outcomes Development Project (LODP) and the College Student Achievement Project (CSAP) teams, supported by the Ministry of, Training, Colleges and Universities (TCU). The results of this work are fourfold, comprised of (a) the principles guiding the development and use of the learning outcomes; (b) the framework used to construct the learning outcomes; (c) common sets of learning outcomes, related learning objectives and topics, and sample assessments for college pre-technology, pre-business, and first-year business diploma-level mathematics courses; and, (d) recommendations for consideration and/or adoption of the materials described in this report.

The report is being distributed to all Ontario colleges for development of mathematics curricula and pedagogy, and examination, revision, and/or review of current mathematics courses. The adoption of these sets of learning outcomes and their related materials have the potential to help students bridge the mathematics learning gap from secondary to college classrooms, strengthen mathematics teaching and learning in Ontario college classrooms, and provide consistent educational opportunities for students transferring between colleges. In these ways, success in student mathematics learning and college programs could improve.

The LODP team would like to acknowledge the contribution of the Ontario colleges to this project. The course outlines they submitted were used extensively in the development of the topics lists and mapping to the learning outcomes. In addition, the input provided by all stakeholders during the feedback process contributed significantly to the final product.

In addition, special thanks go to Tanya Jessup, the technology subject lead from Durham College, and Margaret Dancy, the business subject lead from Conestoga College, for their expert development, guidance, and critique of the learning outcomes. Their knowledge of their subject areas was invaluable to the advancement of the project. Particular recognition goes to Pina Marinelli-Henriques, CSAP Project Manager, and Sarah Brumwell, Research Assistant, for their indispensable expertise and dedication to the project.

Patricia Byers, PhD, Project Lead

Laurel Schollen, CSAP Director

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Chapter 1: Introduction and Background

The Learning Outcomes Development Project (LODP) is a collaborative effort of college faculty dedicated to developing mathematics courses that facilitate the success of students transitioning to college learning. The project received a clear mandate from the Ministry of Training, Colleges and Universities (TCU) as a result of a feasibility study conducted by the College Student Achievement Project (CSAP) in 2013. The primary project goal was to:

- develop learning outcomes for pre-technology, pre-business, and first-year business diploma-level, and college mathematics courses.

The purpose of this goal was to:

- foster smooth mathematics transitions between secondary school and colleges, and
- support the transfer of college students between institutions.

As the project unfolded, it became clear that three additional goals would be addressed. Adoption of a set of learning outcomes for the common use of Ontario's community college system could:

- create high-quality student learning experiences,
- clarify college mathematics curriculum and pedagogy to secondary school mathematics teachers and administrators on a system, rather than institutional level, and
- demonstrate respect for college academic freedom and autonomy.

At all times, the focus of the development of the learning outcomes was to provide a meaningful framework for teaching and learning mathematics that supports a college education.

The LODP team recognizes that subject learning outcomes are part of a fundamental trajectory for students to bridge the gap from secondary to college mathematics learning. This trajectory needs to be seamless, without unexpected obstacles to student learning, while supporting students as they navigate college mathematics. One strategy to achieve these goals is to provide in-coming college students with a common set of learning outcomes that reflect their learning needs. It is with this goal in mind that the learning outcomes package was developed for Ontario colleges. Colleges and college faculty are encouraged to use the learning outcomes, and any and all related materials to support and improve college mathematics teaching and learning.

This document, ***Bridging the Mathematics Gap through Learning Outcomes***, summarizes the activity conducted by the LODP team towards achieving these goals.

Background

The College Mathematics Project (CMP) was a collaborative program of research and deliberation conducted from 2006 to 2011. It focused on the mathematics achievement of Ontario first year college mathematics students. An underlying goal of the project was to support mathematics learning success; and, a fundamental way to achieve this goal was to identify and address factors contributing to mathematics learning from secondary school to college classrooms. In so doing, students could experience a seamless transition in their mathematics learning. This research has garnered the attention

of Ontario's community colleges, the School College Work Initiative (SCWI), the Ministry of Education (EDU), and the Ministry of Training, Colleges and Universities (TCU) to the point where they have participated and provided financial support, thereby sustaining and promoting the project.

Based on the CMP 2011 research results, TCU requested the development of learning outcomes and curricula that would provide direction and structure for Ontario colleges' pre-technology, technology, pre-business, and business mathematics courses. The CSAP team was charged with conducting a feasibility study to determine the plausibility of this task. One major outcome of the study was the establishment of the Learning Outcomes Development Project (LODP) in Fall 2013. The team leading this project was tasked to create a common set of learning outcomes for pre-business, first-year business diploma-level, and pre-technology mathematics courses; these learning outcomes and resource materials would be made available to all Ontario colleges.

Finding Commonalities in College Mathematics Courses

The College Mathematics Project (CMP) 2011 *Final Report* included a qualitative component which examined pre-business, business, pre-technology, and technology mathematics courses. The intent of this examination was to obtain a province-wide perspective of mathematics taught in college classrooms. A key outcome of this examination was the commonality of topics taught in colleges among and between these courses. Conclusions of the study emphasized the importance of numeracy in first semester college mathematics courses. This emphasis was not placed simply on re-teaching Grade 11 and 12 mathematics topics, or earlier in a student's educational journey. Rather, it was recommended that a focus on mathematics taught in a student's education should be directed towards their career goals[1]. That is, mathematics topics such as algebra, ratio and proportion, order of operations, and so forth, should be taught in first semester in contexts that makes sense to student learning and college programs goals. The success of implementing this recommendation would be heightened if colleges could work together to develop and teach the mathematics system-wide[2].

Focusing the Learning Outcomes Development Project

These research results and others were noticed by TCU, which requested the development of a common province-wide assessment tool, and common learning outcomes and curricula for pre-business, business, pre-technology and technology mathematics courses. These projects were seen as an opportunity for colleges to work towards the common goals of student success and retention[3]. In response to this request, CSAP conducted a feasibility study, *Developing Mathematics Assessment, Learning Outcomes and Curriculum for the Ontario College System: Final Report of a Feasibility Study*. The results of this research confirmed the CMP 2011 results that there was a high degree of topic commonality among the pre-business and pre-technology mathematics courses[4]. However, there was less commonality among the business and technology courses. In addition, it was identified that there

[1] G. Orpwood, L. Schollen, G. Leek, P. Marinelli-Henriques, H. Assiri. *College Mathematics Project 2011: Final Report*. (Toronto: Seneca College of Applied Arts & Technology, 2012), pp. 66 & 67.
[2] Ibid, p. 69.
[3] G. Orpwood, E. Brown. *Developing Mathematics Assessment, Learning Outcomes and Curriculum for the Ontario College System: Final Report of a Feasibility Study*. (Toronto: Seneca College of Applied Arts & Technology, 2013), p. 5.
[4] Ibid, p. 25.

was enough diversity within the technology programs to not recommend the development of a set of common learning outcomes and a common curriculum[5]. Therefore, the study recommended to TCU that a common set of learning outcomes be developed for pre-technology, pre-business, and business mathematics courses[6].

Vocational Standards, College Mathematics, and Learning Outcomes

Most Ontario college program standards are comprised of three elements: vocational learning outcomes, essential employability skills, and the general education component. The major goals of these program standards are two-fold: to provide college graduates key vocational skills that will help them find employment in their field of study; and, to provide broader skills and knowledge that will give students flexibility to continue to learn and adapt throughout their working lives[7]. For example, all program standards under the *Essential Employability Skills* standards include a focus on communication, numeracy, critical thinking and problem solving, information management, and interpersonal and personal skills. Students of college programs must demonstrate these standards upon graduation.

The vocational program standards are written in the form of learning outcomes which apply directly to the program. Each college can interpret the learning outcomes with respect to their student and college needs. In this way, TCU respects the autonomy of each college in their interpretation of curriculum and pedagogical matters of a program while ensuring consistency in meeting program criteria. According to the TCU website, "They [the vocational standards] are not simply a listing of discrete skills, or broad statements of knowledge and comprehension. Learning outcomes in the program standards represent culminating demonstrations of learning and achievement[8]."

In addition, the learning outcomes are further sub-divided into *Elements of the Performance*, which define and clarify the learning outcome. While the *Elements of the Performance* may indicate how the student achieves the learning outcome, it is the learning outcome by which the student is evaluated[9].

The CMP 2011 *Final Report* identified mathematics topics from the pre-business and pre-technology TCU program standards that focus on mathematics to compare with the mathematics topics generated from the college course outline analysis. In many program areas, the vocational program standards are to be considered as a comprehensive whole; therefore, notions of mathematics are found within many of the learning outcomes. For example, in the Business Fundamentals Program vocational standards [10], there are eight learning outcomes, some which apply directly to mathematics, while others apply indirectly to mathematics.

On the other hand, an examination of the Electrical Techniques Program vocational standards would find more explicit identification of mathematics topics. For example, there are 16 Vocational Standards Learning Outcomes. The second learning outcome specifically identifies applications of mathematics

[5] Ibid, p. 35.
[6] Ibid, p. 36.
[7] http://www.tcu.gov.on.ca/pepg/audiences/colleges/progstan/intro.html#overview
[8] http://www.tcu.gov.on.ca/pepg/audiences/colleges/progstan/contain.html
[9] Ibid, p. 4.
[10] *Business Fundamentals Program Standard*. TCU 40208 (Toronto: Ministry of Training, Colleges and Universities, 2012).

related to electrical systems with six elements of performance. These learning outcomes identify mathematics in a general way but do not necessarily identify specific mathematics skills. That is, specific mathematics skills describe “Basic Algebra;” however, the mathematics related to “Trigonometry” is not described.

Nonetheless, results from the CMP 2011 *Final Report* and results from the CSAP 2013 *Feasibility Study* identify more detailed mathematics skills that students need to demonstrate for success in their chosen program.

The learning outcome writing model adopted for this project reflects these concepts as set out by TCU and builds on results from the two research studies. Faculty will be familiar with this learning outcome model and subsequent topic mappings, and able to identify how the course learning outcomes are extended to the program learning outcomes.

Bridging the Secondary School/College Gap through Learning Outcomes

The LODP analysis of secondary school and college curricula reveals parallel structures in learning outcomes; i.e., each uses a learning outcome model with defined expectations describing how a student should think, know, say, and do in order to achieve specified learning. A fundamental difference between the learning outcomes models is the terminology used to describe the curriculum framework; that is, Learning Outcomes in the TCU program standards are known as Overall Expectations in the EDU curriculum documents, and Elements of Performance are known as Specific Expectations respectively.

An illustration of how these learning outcome frameworks compare is shown in Figure 1: *Comparing college and secondary school curriculum frameworks*.

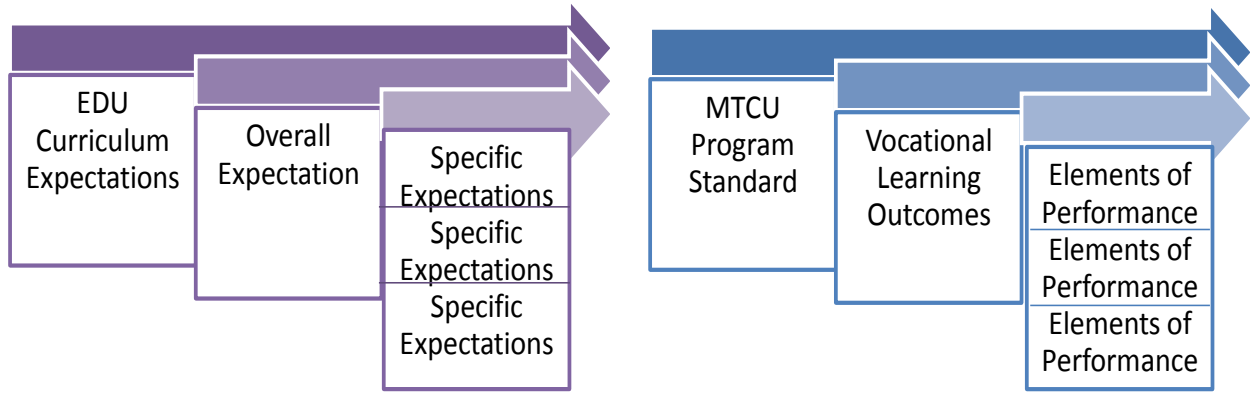

Figure 1: Comparing college and secondary school curriculum frameworks.

The secondary curriculum is divided into strands and within each strand there is an Overall Expectation and the Specific Expectations[11]. The Overall Expectations in the EDU curriculum documents describe in general terms the knowledge and skills expected of students by the end of the course. The Specific Expectations unpack the knowledge and skills in greater detail; they may provide a guide for lesson planning. As with the TCU program standards learning outcomes, achieving each learning outcome is not accomplished in isolation of other learning outcomes[12]. A major difference between the curricular documents is that the college curriculum is based on a program model where the secondary school curriculum is based on a subject model. College faculties are challenged to embed and extend subject material learned in the secondary school classroom into context-based program courses.

The LODP team focused on writing learning outcomes for pre-technology, pre-business, and first-year business diploma-level mathematics courses that follow a framework similar to those used in the TCU program and EDU curricular documents. These learning outcomes will provide a framework for the mathematics learning students require to be successful in their chosen career path.

[11] This framework is discussed in *The Ontario, Grades 1-8, Mathematics* (Toronto: Ministry of Education, 2005), p. 8; *The Ontario, Grades 9-10, Mathematics* (Toronto: Ministry of Education, 2005), p. 8; and, *The Ontario, Grades 9-10, Mathematics* (Toronto: Ministry of Education, 2005), pp. 11-12.

[12] *The Ontario Ministry of Education, Grades 11 and 12*, Mathematics (Toronto: Ministry of Education, 2007), p. 11.

Chapter 2: Work Plan and Methodology

The LODP work plan was organized into seven phases and is summarized below. Within this summary is a methodology used by the LODP writing team to generate sets of meaningful learning outcomes and ancillary materials designed for examination, comparison, and/or adoption for college mathematics classrooms. Included in the ancillary materials are the principles used to guide the writing process and the learning outcomes model.

Summary of the LODP Work Plan

Phase 1: Establishing the Learning Outcomes Development Project

- The Advisory Committee for the LODP and the Assessment Development Project (ADP) was assembled. The ADP and LODP are "companion" projects to the main CSAP project and as such report to the CSAP Steering Committee. The ADP/LODP Advisory Committee is responsible for providing guidance to the CSAP Project Team on the overall strategic direction of both projects. Membership included representatives from the secondary school boards, college community, TCU, EDU, and the CSAP team.
- The Terms of Reference, governance structure, and reporting mechanisms to college operating groups, including the Ontario Colleges Mathematics Council (OCMC), Heads of Interdisciplinary Studies (HoIS), Heads of Technology (HOT), Heads of Business (HOB), and Coordinating Committee of Vice Presidents Academic (CCVPA), were developed.
- Colleges were contacted to submit course outlines and sample assessments from their technology foundations, business foundations, and/or first-year business diploma-level mathematics courses.

Phase 2: Developing the course outline analysis framework

- A framework for analyzing course outlines and learning outcomes was developed and applied to the materials collected from the colleges.
- Pivot tables were constructed allowing the LODP writing team to analyze content based on mathematics topic(s), verb(s), college(s), and learning outcome(s).

Phase 3: Developing the learning outcomes writing framework

- Learning outcome topic areas, including mappings back to individual college course learning outcomes and objectives, were identified.
- Eight principles were developed to guide and facilitate the writing of the learning outcomes. They provided rationale for writing the learning outcomes to reflect the mandate of the CSAP project.
- A three-tiered framework was developed as a platform to design the learning outcomes and integrate the topic lists generated from the college course outline analysis. This framework is based on concepts related to learning outcomes writing as discussed in Learning Outcomes Primer by V. Lopes[13]. This framework can be used to design a course or augment resources in order to enhance classroom teaching. Features of this framework are:
    - learning outcomes

[13] V. Lopes. (2013). *Learning Outcomes Primer*. (Toronto: Seneca College of Applied Arts & Technology).

- learning objectives
- sample assessments

These features are characterized in Figure 2**:** *The learning outcomes framework*.

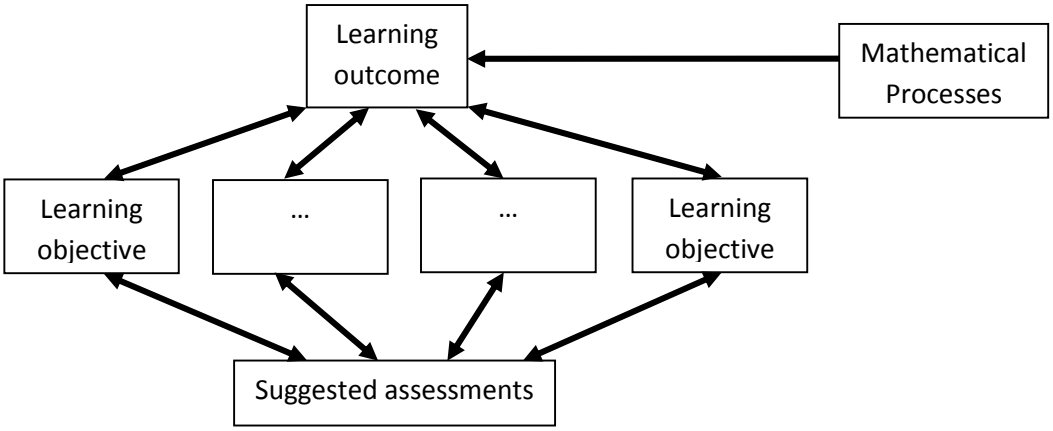

Figure 2: The learning outcomes framework.

Phase 4: Writing the learning outcomes

- Business mathematics and technology foundation mathematics college faculty leads reviewed, validated, and analyzed course outlines for mathematics topics similarities and omissions. Additional mapping back to individual college course learning outcomes and objectives was conducted. Topic lists for technology foundations, business foundations, and first-year business diploma-level mathematics courses were generated.
- The learning outcomes writers used information gathered from the course outlines analysis to generate the framework for the learning outcomes and write the learning outcomes. Common learning outcomes for technology foundations, business foundations, and first-year business diploma-level mathematics courses were written by the LODP writing team.

Phase 5: Validating and revising the draft outcomes

- Draft learning outcomes and a feedback questionnaire were distributed to stakeholder groups (individual colleges, ADP/LODP Advisory Committee, HOT, HOB, OCMC, HoIS, CCVPA).
- The feedback was analyzed, revisions made to the learning outcomes, and the final report written.

Phase 6: Release of final learning outcomes

- Packages containing the final report, the learning outcomes, and communiqué are distributed to the stakeholder groups.

Phase 7: Confirming acceptance of common learning outcomes

- Each college is requested to sign off the learning outcomes in a process similar to the one implemented by TCU with respect to adoption of college program standards.

Principles Used in Writing the Learning Outcomes

It was determined early in the writing process that a set of guidelines was needed to ensure quality of the final product of the project. These guidelines became the principles upon which stakeholders' interests were captured; a writing framework based on educational research and practice was developed; meaningful linkages between the secondary school and college curricular documents were framed; and, respect for the colleges' ability to provide quality education for in-coming students was to be demonstrated.

1. **The primary purpose of the learning outcomes is to provide structure to college pre-technology, pre-business, and first-year business diploma-level mathematics courses.**

Learning outcomes provide a necessary framework to a subject or course. They are the foundation of a teaching/learning opportunity and will exemplify course content expectations. They are student focused and guide the instruction needed for students to learn a particular topic. Since learning outcomes are measurable, they also provide a framework for the assessment model used to evaluate student learning. As stated in the *Learning Outcomes Primer*, "Learning Outcome Statements clearly state the knowledge, skills or attitudes that students will be able to demonstrate upon the successful completion of a subject [14]." This resource underscores the importance of learning outcomes in course development and delivery, while allowing individual college autonomy to interpret how these learning outcomes are to be achieved.

The centre of any college course is its learning outcomes. A well written learning outcome is considered to be SMART: specific, measurable, attainable, relevant, and timely. Generally, learning outcomes have three components: an action verb that is student-centred and measurable; content pertinent to the course; and, a context that supports the college program. The learning outcomes for the pre-business, business, and pre-technology mathematics courses reflect these three dimensions. These dimensions are: taxonomy, topics and applications.

2. **Faculty (team, department, faculty, or institutional approach) will be able to independently select those learning outcomes pertinent to their college foundation and diploma program(s).**

This project recognizes the unique needs and local conditions colleges have with the respective communities and businesses that they serve. The mathematics curriculum adopted by a college is based on a number of factors including its admission criteria and the knowledge students bring to college, to name a few. Colleges also support a local community or business sector needs through individualized programs[15]. The CSAP LODP respects their decision to accept none, some, or all of the learning outcomes generated from this project[16].

3. **The learning outcomes reflect the elementary and secondary school mathematical processes.**

The elementary and secondary school mathematics curriculum expectations are framed by seven mathematical processes: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing and communicating. These processes help guide

[14] *Ibid*., p. 1.
[15] Orpwood (2013), *op. cit*., p. 27.
[16] *Ibid*., p. 28.

student learning in each mathematical concept they are learning. The processes are interconnected, and are integral to the learning of mathematics. When students learn, apply, and extend their mathematical thinking, they are applying these mathematical processes in a deliberative way. The mathematical processes also mirror the "Key Cognitive Strategies" identified as one of four categories of College Knowledge[17]; e.g., analysis, interpretation, precision and accuracy, problem solving and reasoning.

4. **The learning outcomes will consider the three cognitive domains described in TIMSS.**

The LODP adopted the framework utilized by the Third International Mathematics and Science Study (TIMSS)[18]. This model utilized two dimensions: content and performance expectations. The performance expectations are organized within three cognitive domains – knowing, applying, and reasoning. TIMSS characterizes each domain in terms of measurable tasks that students are expected to perform.

- Knowing: recall, recognize, compute, retrieve, measure, classify/order
- Applying: select, represent, model, implement, solve routine problems
- Reasoning: analyze, generalize/specialize, integrate/synthesize, justify, solve non-routine problems

The measureable tasks for each cognitive domain reflect elements of Bloom's Taxonomy – remembering, understanding, applying, analyzing, evaluating, and creating[19]. They also reflect elements of the mathematical processes described in the elementary and secondary school mathematics curricula. Learning outcomes for college pre-technology, pre-business and business mathematics should reflect these elements.

The TIMSS 2011 assessed students in the fourth- and eighth grades. The study targeted percentages of time each cognitive domain spent assessing students at each grade level. Grade 4 students were expected to spend 40% of the testing time on knowing and applying; 20% of the testing time on reasoning. These proportions compare with those of Grade 8 students where they were expected to spend less time utilizing the knowing cognitive domain (i.e., 35%) and more time on the reasoning domain (i.e., 25%). Students in college programs are required to employ higher order thinking as they develop sophisticated understanding of how mathematics should be applied in program expectations. The LODP writing team needed to consider how to incorporate this level of mathematics understanding into the learning outcomes.

5. **Common learning outcomes will reflect notions of independent and collaborative learning, metacognition, and an appreciation of mathematics used in our world and industry.**

These notions are reflective of the mathematical processes, such as problem solving, reasoning and proving, reflecting, and communicating. Examples of these learning outcomes are:

- Explain the value of mathematics and the applications of mathematics used in business/technology.
- Work individually and co-operatively to solve mathematics problems related to technology and science.

[17] Orpwood (2011). Op cit. p. 70.

[18] http://timssandpirls.bc.edu/timss2011/downloads/TIMSS2011_Frameworks-Chapter1.pdf

[19] http://www.techlearning.com/studies-in-ed-tech/0020/blooms-taxonomy-blooms-digitally/44988

6. **Learning outcomes for pre-technology mathematics courses should differ from learning outcomes for pre-business mathematics courses. And, learning outcomes for pre-business mathematics courses should differ from learning outcomes for business courses.**

The CSAP feasibility study identified, with varying degrees of acceptance, a set of topics common to the pre-technology and pre-business programs. These topics are: order of operations, fractions, decimals, percentages, ratio and proportion, algebra, and exponents[20]. How these numeracy topics are applied will differ with respect to their contexts. Learning these mathematical concepts will be enhanced and made more relevant to the student if the concepts are taught reflective of the students' program of study.

7. **The foundation learning outcomes will reflect knowledge and reasoning taught in secondary school curriculum and not beyond.**

The LODP acknowledges the value of a secondary school mathematics education for students entering college programs. However, CMP research revealed that many students struggle in first semester college mathematics. Therefore, some students may benefit from taking a college foundation mathematics course. It is believed that this course should provide students with opportunities to learn mathematics in contexts that prepare them for college diploma programs. The learning outcomes for the foundation courses will not reflect diploma level mathematics.

An ongoing review of the secondary school mathematics curriculum guided the LODP writers when developing the framework for the learning outcomes. For example, 'absolute value' is not taught in the secondary school curriculum; therefore, this concept was not included in the foundation or diploma learning outcomes.

8. **The learning outcomes will reflect the TCU program standards.**

The TCU has program standards for business programs in general and specifically (e.g., business administration, marketing management), pre-business program (i.e., business fundamentals), and a pre-technology program (e.g., motive power fundamentals, mechanical techniques). As part of the program proposal process, a mapping of the proposed program learning outcomes to existing approved program learning outcomes, as published in a standard or in a "program description" (where there are no published program standards) must be completed. These standards include vocational and essential employability skills, and are written as learning outcomes. They are written in terms of learning outcomes and identify specific indicators of performance. The learning outcomes and mapping of courses to the program learning outcomes is reviewed by the Credential Validation Service (CVS) to ensure that the proposed program credential and title is appropriate before applications are approved by TCU. In this way, the college is to provide the highest quality of education described in the program standards.

[20] Orpwood (2013). *Op.cit*., pp. 24 & 25.

The standards include expectations that students will graduate with skills and knowledge related to mathematics.

Examples:

- ***Pre-technology (Mechanical Techniques):*** Vocational Learning Outcome 8: perform routine technical measurements accurately using appropriate instruments and equipment.
- ***Pre-business:*** Vocational Learning Outcome 5: perform basic accounting procedures and financial calculations to support the operations of an organization.
- ***Business:*** Vocational Learning Outcome 8: use accounting and financial principles to support the operations of an organization.

Generating the Topic Lists

The college course outlines were actively mined for topics most likely taught in a particular course in order to frame the learning outcomes. In general, the topics identified most frequently in the analysis were included in the learning outcomes mapping. However, at times, some topics were included as deemed by the subject experts.

Topics identified most frequently in the pre-technology course outlines are: algebra, basic arithmetic, business applications, equations and linear equations, graphing, percentages, and ratio and proportion.

The proportion of pre-technology mathematics topics identified less frequently in the college course outlines are shown in Figure 3: *Percentage of topics LEAST identified in pre-technology mathematics courses*.

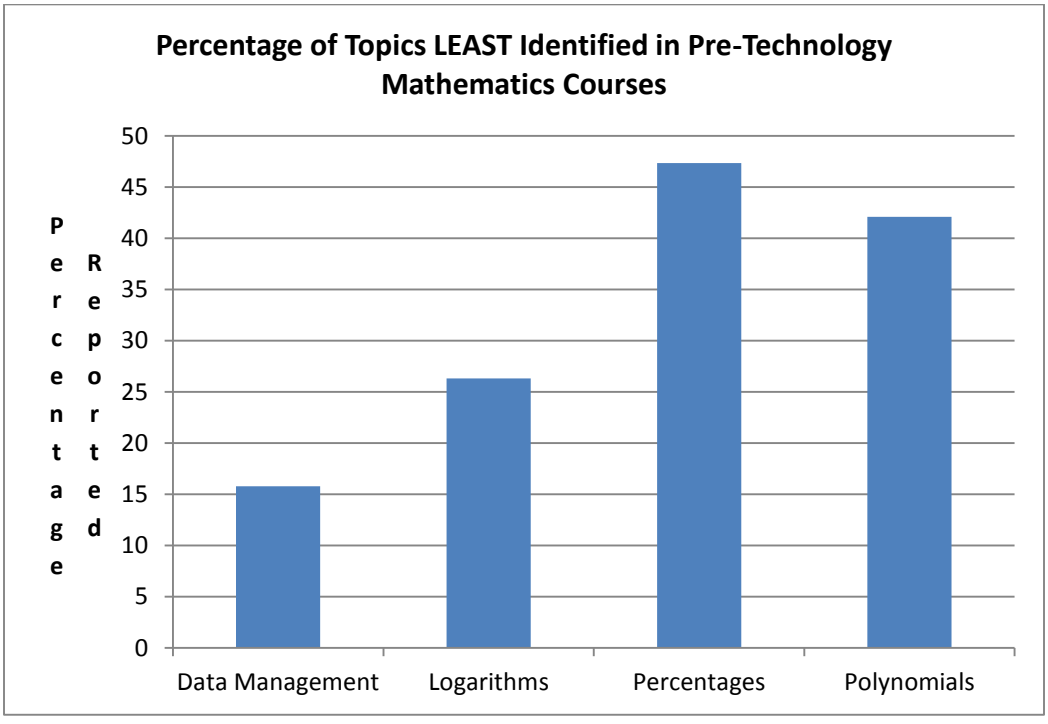

Figure 3: Percentage of topics LEAST identified in pre-technology mathematics courses.

The pre-business mathematics topic analysis identified the following topics as those included in this course by a majority of colleges. They are: algebra, basic arithmetic, numeration and decimals, business concepts, equations and exponents, and problem solving. The bar chart in Figure 4: *Percentage of topics LEAST identified in pre-business mathematics courses* illustrates those topics that few colleges include in their course outlines.

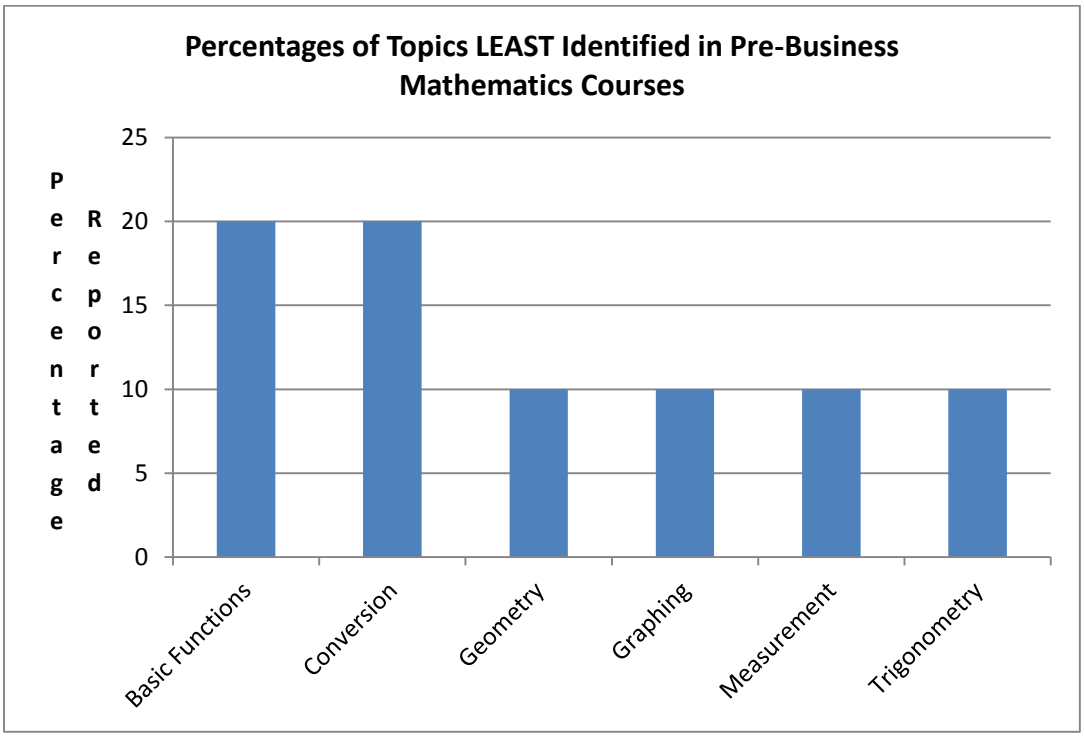

Figure 4: Percentage of topics LEAST identified in pre-business mathematics courses.

Topics which appeared most frequently in the business course outlines are: algebra, basic arithmetic, business applications, equations and linear equations, graphing, percentages, and ratio and proportion. The proportion of business mathematics topics identified less frequently in the college course outlines are shown in Figure 5: *Percentage of topics LEAST identified in business mathematics courses*.

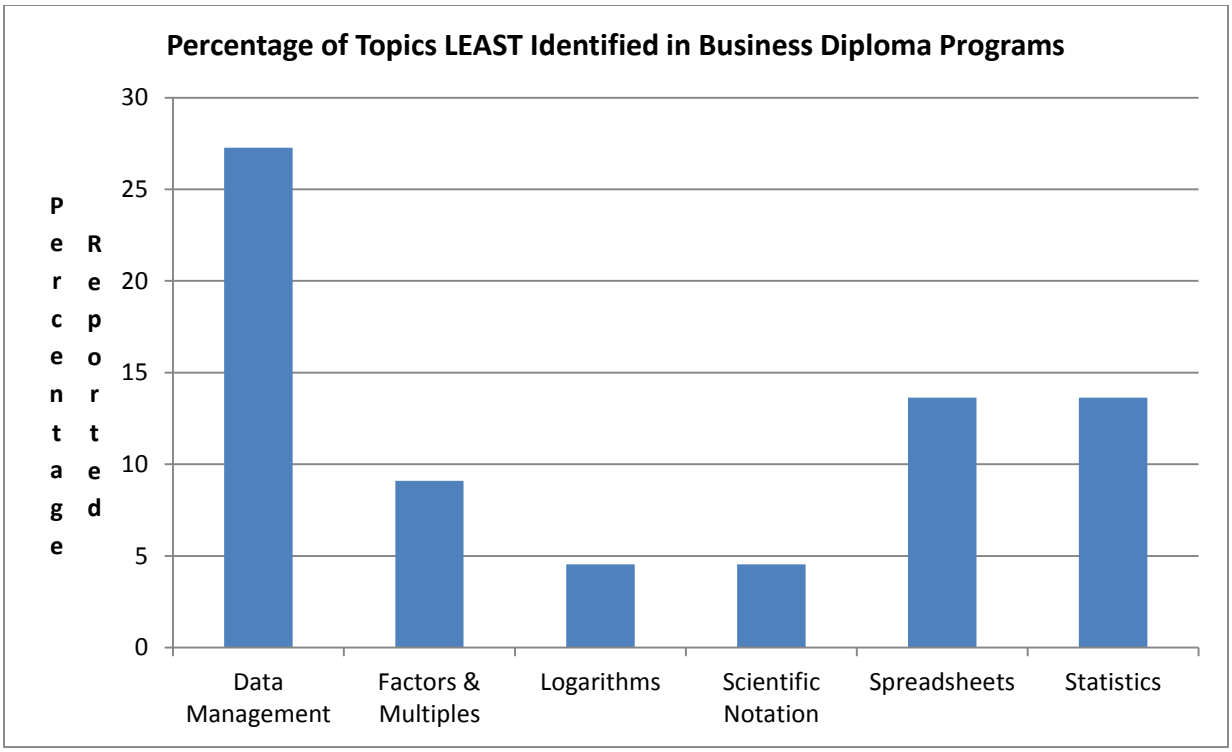

Figure 5: Percentage of topics LEAST identified in business mathematics courses.

In general, it is interesting to note the mathematics topics that some colleges include in their foundational courses. Some colleges expect students to come to college technology and business with a sound set of mathematics skills including scientific notation, which is not taught in the secondary school mathematics curriculum, and functions and logarithms, which are not taught in MAP4C: Foundations for College Mathematics. These topics are not included in the LODP learning outcomes package in keeping with Principle 7: *The foundation learning outcomes will reflect knowledge and reasoning taught in secondary school curriculum and not beyond*.

Chapter 3: The Learning Outcomes Writing and Feedback

The learning outcomes were written consistent with the overarching principles using a three dimensional model (taxonomy, topic and application), as described in Chapter 2 and shown in Figure 6: *Modelling the learning outcomes writing*.

The verb in each learning outcome was obtained from a taxonomy consisting of three categories suggested in the TIMSS report: knowing, applying, and reasoning. This taxonomy of verbs also coincides with the taxonomy used to categorize test items in the Assessment Development Project (ADP), a separate project that is charged with developing a common post admission mathematics skills assessment for the college system. The intent was to draw parallels between the two projects in order to link the test items with the learning outcomes. The verbs also reflected the mathematical processes used in the secondary school curriculum. In an effort to bridge the gap from secondary to college mathematics learning, Principle 3: *The learning outcomes reflect the elementary and secondary school mathematical processes*, was incorporated into the design of the learning outcomes and learning objectives. Mathematical processes such as problem solving, reasoning and proving, reflecting on a solution, making connections with real-world mathematics problems, representing using models and formulas, and communicating the mathematics are incorporated within the learning outcomes and learning objectives. In this way, approaches of teaching and learning mathematics applied in the elementary and secondary school curriculum expectations were extended to the postsecondary panel by way of the college mathematics common learning outcomes. It is believed that teachers in both educational sectors will be able to trace student mathematics learning and their related concepts from secondary school to college classrooms. Students may also find connections to prior learning if colleges adopt the common learning outcomes into classroom teaching. In this way, the transition from secondary school to college mathematics learning could be less problematic for students.

The subject used in each learning outcome was based on (a) topic(s) identified from mathematics in general, and the topic lists generated from the college course outlines. The applications came from the area of technology or business and served to ground each learning outcome in the discipline for which they were written.

Each learning outcome was unpacked through learning objectives. These objectives are two-dimensional, each containing and a sub-topic detailing facets of the main topic category. Together, the learning outcomes and the learning objectives form the common product for framing the college pre-technology, pre-business, and first-year business diploma-level mathematics courses.

A third tier of the model, the topics, was then constructed, but not as a conclusion of the learning objectives. That is, the topics did not evolve from the learning objectives. Rather, the topics were originally generated from the college course outlines and mapped back to the appropriate learning outcomes and learning objectives. The topics are one-dimensional and do not require a verb or an application. The topic list was supported by a review of three documents: (a) CMP 2011 *Final Report*; (b) the document, *Developing Mathematics Assessment, Learning Outcomes and Curriculum for the Ontario College System: Final Report of a Feasibility Study* CSAP 2013; and, (c) the review of college course outlines in Fall 2013. The initial topic lists were revised until a final working framework was obtained. If a particular topic was not well represented through the review of the college course outlines, it was not included in the final topic list. Additional topics were included as determined by the learning outcomes subject experts and from recommendations of the ADP/LODP Advisory Committee. The relationship

between a learning outcome, a learning objective, and a topic is shown in Figure 6: *Modelling the learning outcomes writing*.

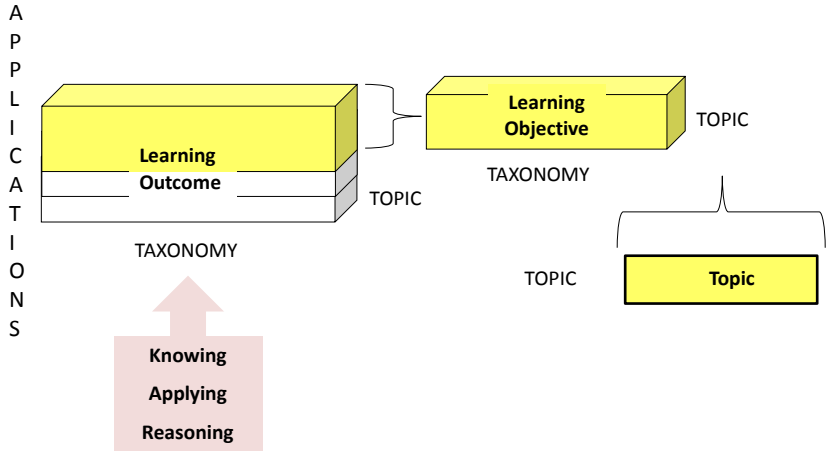

Figure 6: Modelling the learning outcomes writing.

Mathematics learning is not simply based on a list of topics but rather how students integrate knowledge and apply it to real-world contexts. Many college faculty would agree that students are motivated to learn the language of their chosen career, apply the mathematics in a context to recognize its relevancy, use a calculator appropriately to perform complex mathematical operations, and develop an ability to recognize the similarities and differences between formulas. In addition, the learning outcomes reflect the need to help students develop a focus for their learning and a tenacity to work through a problem. To this end, writing the learning outcomes was achieved by answering the question, "What does the student need to know, think, say, and do in order to complete this course?" One way to effectively address this question was to incorporate elements of a problem solving approach, a mathematical process utilized in the elementary and secondary school curricular documents, into the learning outcomes writing. Additional EDU mathematical processes provided the framework for the learning outcomes writing, thereby connecting the secondary school curriculum to the college learning outcomes.

Feedback and Analysis

In Phase 5 of the project work plan, a mechanism was established and implemented to inform the stakeholders of the work performed with respect to the learning outcomes writing, and to gain feedback to endorse or improve the materials. A learning outcomes package was created that included an overview of the project, the learning outcomes, and sample assessments that could be used to assess the learning outcomes. Two appendices were included: (a) a glossary of terms used in the learning outcomes and learning objectives; and, (b) the learning outcomes and learning objectives with a topic mapping. Questions were provided in the learning outcomes package to obtain their input into the learning outcomes writing process. The questions were posed with respect to each area mandated by the project: the pre-technology, pre-business, and first-year business diploma-level mathematics courses.

Feedback was represented by all stakeholder groups; 27 individuals responded to the questionnaire. In particular, 21 out of 24 colleges provided input including the two francophone colleges. There was

opportunity for overlap in the categories identifying the respondents. Therefore, all colleges could have been represented; respondents may have felt it inappropriate to select a college as an identifier. For example, one category of respondents was the CSAP Advisory Committee. A respondent who selected this identifier may have also been from a community college but may not have believed that he/she was also representing college feedback.

Overall, the feedback regarding the Glossary suggested that it was confusing and did not contribute towards an understanding of the learning outcomes. Therefore, the language was simplified to the extent that the Glossary was not required for interpreting the package.

In general, feedback regarding the learning outcomes was positive; respondents liked the learning outcomes and the problem solving approach. Feedback identified that the learning outcomes:

- address the issue of curriculum in terms of what needs to be taught in a particular course;
- address the issue of pedagogy in terms of how the learning needs to take place through the mathematical processes embedded in the taxonomy of verbs;
- provide a framework for a baseline set of numeracy skills; and,
- serve as a benchmark for minimum mathematics skills.

Additional input was categorized into four themes.

Theme 1: Impact on teaching and learning

According to respondents' feedback, the learning outcomes supported college mathematics teaching and learning in five ways.

1. The learning outcomes could be used to develop a business or a technology foundational, or a first-year business diploma-level course. For all colleges to consider this option would be a step forward towards establishing a common course outline in these subject areas.

2. The learning outcomes establish a curriculum framework; and, this framework is based on a problem solving approach as opposed to a topic approach. Respondents are prepared to move college mathematics education forward, away from a subject based model. It is evident there is a need to refocus college teaching and learning towards a model that is focused on solving real world discipline-based applications. In this way, colleges are willing to consider a different model for structuring classroom curriculum pedagogy.

3. Some respondents felt that the learning outcomes could be used to evaluate current programming. They can be used to measure a particular college's learning outcomes and have their learning outcomes validated or put under scrutiny for improvement.

4. The learning outcomes provide a common platform cross-provincially for these courses. This was an important consideration that evolved from the Feasibility Study. Student transferability of credit and inter- and intra-college support for a mathematics curriculum with a common goal are but two major benefits support student mathematics learning should colleges adopt these learning outcomes.

5. The learning outcomes writing process was considered by the respondents as a viable model with which to adopt for their own learning outcomes writing. There was evidence from the feedback that colleges are not just looking for a set of learning outcomes to adopt in a wholesale manner. They are also looking for a methodology with which to develop their own learning outcomes. In addition, by understanding the process by which the learning outcomes were developed strengthens one's understanding of the learning outcomes themselves and their role in teaching and learning mathematics.

Theme 2: Numeracy

Question #2 of the Feedback Questionnaire asks, "In which ways do the learning outcomes support numeracy at your college?" All respondents answered this question; reviewers at one college felt that overall, the document encompass the important aspects of numeracy. However, there was an underlying perspective that the outcomes did not go far enough in the foundation mathematics courses. Some feedback went as far to suggest a learning outcome be inserted into the foundational learning outcomes. This learning outcome focuses on supporting mental arithmetic and facility in the pre-technology and pre-business learning outcomes. This learning outcome highlights the importance of being able to perform mental arithmetic and not be calculator dependent for fundamental calculations. In fact, this arithmetic skill is critical in estimation and determining whether or not an answer, displayed on a calculator or not, is reasonable.

> *Demonstrate the skill to quickly and accurately perform basic mathematical operations (for example: adding, subtracting, multiplying, dividing, and simplifying basic integer, rational and radical numerical and algebraic expressions) without the aid of technology.*

As a learning objective, this statement addresses the importance of demonstrating basic numeracy skills; of being able to calculate positive and negative integers using the four basic arithmetic functions, basic fractions, and simplify radicals using mental arithmetic. This learning objective also supports the Essential Employability Skill, numeracy, so critical in the TCU program standards. This would mean that students would need to be able to utilize the fundamental arithmetic operations without dependency on a calculator.

This suggestion was incorporated into the pre-technology learning outcome #4:"At the end of this course, the student will be able to apply an operation, method, or problem solving strategy to solve or verify the solution to a technology-based problem," and into the pre-business learning outcome #3: "At the end of this course, the student will be able to apply an operation, method, or strategy to solve a business problem."

The use of technology is not discouraged in this learning objective. Indeed, mathematical processes such as selecting tools and computational strategies as opportunities to use technology to perform complex calculations, model relationships, display trends, and so forth, are addressed in the learning outcomes. See, for example, Pre-Business Learning Objective #2.4. "Select technology-based or non-technology-based computational tools as needed to explore, display, manipulate, and present data in a variety of ways; e.g., business calculators, spreadsheets, timelines, schedules, graphs, and on-line calculators such as mortgage calculators and student loan repayment loan calculators from national student loan websites, and currency calculators."

Theme 3: College autonomy

The second principle considered in the learning outcomes writing featured the importance of respecting colleges for their autonomous nature and ability to respond to the educational needs of industry, and their local and regional communities. The learning outcomes and assessments were always presented as tools that would be available to colleges for their independent use. However, the feedback from the learning outcomes package suggested a tension between prescribing a common curriculum and the need for structured topics among respondents. Some respondents were very concerned that a curriculum model was being presented in a wholesale manner to be adopted by the college system. Others wanted more detail in the learning outcomes package to the point where extensive topic lists were suggested. While the learning outcomes with the topic list mappings were placed in appendices, these same respondents wanted to see them in the front matter in order to provide more direction to curriculum and pedagogy. In order to reconcile this tension, the LODP team resolved to maintain the position to move from a "textbook" topic approach and to adopt a problem solving approach, thereby allowing colleges to mould their topic lists according to their program and subject needs. An outcome of this reconciliation was to ensure that the topic lists were (a) generated from mining the colleges' course outlines, thereby honouring the work they submitted; (b) mapped back to the learning objectives; and, (c) written as one-dimensional topics. The two-tiered model as learning outcomes and learning objectives is still considered the final work of this project and consists of the final submission. Colleges are welcome to review the topic lists mapped to the learning objectives and adopt them as needed.

Theme 4: Assessments

The sample assessments for the pre-technology, pre-business, and first-year business diploma-level courses were deemed to be just that, suggested. In fact, in the preamble to the draft learning outcomes it was stated that, "these [assessment] practices are left to the discretion of the colleges." However, some respondents interpreted this segment of the learning outcomes package as prescribed evaluative practices and limited to those identified in the package. In this way the assessments were misunderstood by many respondents in the feedback. The assessments were written and designed to be examples reflecting real-world applications thereby providing a link to college mathematics. As a result of the feedback, it was determined that the list of sample assessments did not reflect how they in fact assessed the learning outcomes. Therefore, the list was removed; one assessment was identified; and, this assessment was mapped back to the learning outcomes in a general sense.

Recommendations coming out of the Feedback

Respondents to the feedback questionnaire recommended promoting the learning outcomes to the college mathematics community. It was clear from the responses that activity surrounding the project should continue towards achieving an understanding of the learning outcomes, how they can be implemented and assessed, and resources to facilitate their adoption and use. These suggestions include:

- Engaging in cross-panel discussions with secondary teachers of the grade 11 college and grade 12 college courses around assessment of the pre-tech and pre-business courses in particular;
- Implementing and evaluating learning outcomes through local and province-wide workshops;
- Sharing related resources and best practices in forums and on provincial mathematics websites such as the OCMA; and,
- Examining and developing alternate ways to assess learning outcomes.

Chapter 4: The Learning Outcomes

The learning outcomes with their respective learning objectives for pre-technology, pre-business, and first-year business diploma-level mathematics courses are presented here in a series of tables. Following each set of proposed learning outcomes is an example of an assessment strategy in the form of a project that students could complete in order to achieve the learning outcomes. Each assessment is mapped back to the respective learning outcomes. The assessments are but one way to meet the recommended learning outcomes. The LODP team recognizes other assessment practices that could be used to meet learning outcomes in either formative or summative forms; e.g., quizzes, portfolios, capstone projects. However, the projects provided here are seen as frameworks for opportunities to engage students in collaborative, interactive problem solving activities. Colleges are encouraged to consider other forms of assessments and evaluative tools in order for students to achieve their learning goals in their program of choice.

Each college is respected for their autonomy to apply individual curricular and pedagogical perspectives at their institution. Particular attention was not given to addressing college classroom assessment or evaluation practices. These areas are considered the purview of the college program and/or faculty member. Therefore, no recommendations are given for specific textbooks, syllabi or schedules of content, teaching practices or pedagogies, college views, and evaluation or assessment practices. Rather, these practices are left to the discretion of the colleges. It is only once these or any other learning outcomes have been adopted that educators can plan the best method of instruction and identify the appropriate resources for student assessment and evaluation.

In Appendices A, B, and C, the learning outcomes and learning objectives are provided with a mapping to the topics mined from the course outlines submitted by the colleges. The topics are one-dimensional and are not meant to be prescriptive. Rather, the topics represent possible mathematics taught in Ontario colleges.

Pre-Technology

LEARNING OUTCOMES	LEARNING OBJECTIVES
1. At the end of this course, the student will be able to mathematically analyze technology-based problems in order to select an operation, method, or strategy.	1.1. Interpret numbers given in a problem. 1.2. Represent mathematical relationships using (a) diagram(s) and/or sketch(es). 1.3. Apply the Imperial and metric (SI) systems to describe measured dimensions, such as length, area, and volume. 1.4. Predict a reasonable answer to the problem, including the units of measure. 1.5. Connect technology-based problems to real-world contexts.
LEARNING OUTCOMES	**LEARNING OBJECTIVES**
2. At the end of this course, the student will be able to select an operation, method, or strategy to solve technology-based problems where there is a known procedure, algorithm, or method of solution.	2.1. Evaluate the use of proportions as a strategy for solving problems involving relationships between ratios. 2.2. Select the algebraic equation(s) needed to solve problems. 2.3. Determine if a function and its graph can be used to interpret and solve problems. 2.4. Determine whether geometric and trigonometric properties and laws can be used as a strategy to solve problems.
LEARNING OUTCOME	**LEARNING OBJECTIVES**
3. At the end of this course, the student will be able to model technology-based problems using linear relationships, graphs, and geometric and trigonometric representations.	3.1. Analyze data using linear relationships and their graphs to predict and solve for unknown values. 3.2. Apply geometric and trigonometric formulas and properties to design models representing problems.
LEARNING OUTCOMES	**LEARNING OBJECTIVES**
4. At the end of this course, the student will be able to apply an operation, method, or problem solving strategy to solve or verify the solution to a technology-based problem.	4.1. Select an operation, method, or problem solving strategy. 4.2. Apply the relationships between decimals, fractions, ratios, and percentages.

	4.3. Solve and predict unknown values using proportional relationships. 4.4 Demonstrate the skill to quickly and accurately perform basic mathematical operations (e.g., adding, subtracting, multiplying, dividing, and simplifying basic integer, rational and radical numerical and algebraic expressions) without the aid of technology. 4.5. Solve linear equations in one variable using principles of algebra involving positive and negative integers. 4.6. Manipulate equations and formulas used in technology-based problems using the properties exponents and radicals. 4.7. Solve problems involving two- and three-dimensional figures using geometric and trigonometric formulas and properties.
LEARNING OUTCOMES	**LEARNING OBJECTIVES**
5. At the end of this course, the student will be able to justify the solution to a technology-based problem to ensure its accuracy and validity.	5.1. Review the solution to a problem for reasonableness and accuracy. 5.2. Test the solution to a problem by inputting the found value into the original algorithm. 5.3. Express the solution according to the numerical expectations and context of the problem, including the correct units of measure.

Mapping of a Sample Assessment to Pre-Technology Learning Outcomes

Sample Assessment: Identify an architectural structure (e.g., church, gate, courtyard) and take measurements in SI units. Calculate perimeter and area for at least five different polygons in this structure. Calculate all unknown angles in degree measure.

LEARNING OUTCOMES	SAMPLE ASSESSMENT MAPPING
1. At the end of this course, the student will be able to mathematically analyze technology-based problems in order to select an operation, method, or strategy.	Identify all polygons in the selected structure. Ensure there are five different types of polygons. The measurements will be in rational numbers taken to one decimal place and in metres. Use a measuring instrument to determine all obtainable lengths of sides. Note the longest lengths in each polygon with the understanding that the longest length in a triangle will be the largest number.
2. At the end of this course, the student will be able to select an operation, method, or strategy to solve technology-based problems where there is a known procedure, algorithm, or method of solution.	Have available formulas to calculate the perimeter and area of these polygons and a calculator recommended by the program.
3. At the end of this course, the student will be able to model technology-based problems using linear relationships, and geometric and trigonometric representations.	Sketch each polygon with the known values identified and the unknown values labelled with a variable. Identify any right angles in triangles and quadrilaterals, if it exists.
4. At the end of this course, the student will be able to apply an operation, method, or problem solving strategy to solve or verify the solution to a technology-based problem.	Input all known values into the formulas and calculate the unknown values. Use the principles of algebra to rearrange and solve the formulas for any unknown variables. Use rounding rules to determine answers to within one decimal.
5. At the end of this course, the student will be able to justify the solution to a technology-based problem to ensure its accuracy and validity.	Check the units of measure. Complete the sketches with the answers found and make a visual check of the sketches to ensure that the lengths are reasonable, e.g., the longest side of any triangle is the largest number in the lengths measured or calculated. Compare all numbers to ensure consistency and reasonableness.

Pre-Business

LEARNING OUTCOME	LEARNING OBJECTIVES
1. At the end of this course, the student will be able to analyze information from business-based mathematics problems to identify a problem solving strategy.	1.1. Identify and define terminology needed to solve a business-based mathematics problem. 1.2. Identify units of measure and conversions found in the problem. 1.3. Put the mathematics into a context that reflects a business formula, equation, or procedure.

LEARNING OUTCOME	LEARNING OBJECTIVES
2. At the end of this course, the student will be able to select the correct formula(s), and computational tools and strategies to solve basic business-based problems.	2.1. Determine a problem solving strategy. 2.2. Determine the formula, equation, or procedure needed to solve a problem. 2.3. Explain how a formula is connected to the solution of a business-based problem. 2.4. Select technology-based or non-technology-based computational tools as needed to explore, display, manipulate, and present data in a variety of ways; e.g., business calculators, spreadsheets, timelines, schedules, graphs, and on-line calculators such as mortgage calculators and student loan repayment loan calculators from national student loan websites, and currency calculators.

LEARNING OUTCOME	LEARNING OBJECTIVES
3. At the end of this course, the student will be able to apply an operation, method, or strategy to solve a business problem.	3.1. Demonstrate the skill to quickly and accurately perform basic mathematical operations, e.g., adding, subtracting, multiplying, dividing, and simplifying basic integer, rational and radical numerical and algebraic expressions, without the aid of technology. 3.2. Express numbers in the form of fractions, decimals, and per cents to solve problems related to ratio and proportion. 3.3. Solve equations using fundamental principles of arithmetic involving positive and negative integers.

	3.4. Solve business-based formulas using the properties of exponents and radicals. 3.5. Solve single variable algebraic equations in real world business problems using the principles of algebra.

LEARNING OUTCOME	LEARNING OBJECTIVES
4. At the end of this course, the student will be able to model real world business problems using linear relationships.	4.1. Plot points in the four quadrants of the Cartesian plane using integral ordered pairs. 4.2. Represent business relationships with bar graphs and circle graphs. 4.3. Represent linear relationships in a business analysis context with tables of values, slope-intercept form, standard form, and x- and y-intercept form.

LEARNING OUTCOME	LEARNING OBJECTIVES
5. At the end of the course, the student will be able to justify the solution to a business problem to ensure its accuracy and validity.	5.1. Review steps of the solution for accuracy, attempting a different strategy if incorrect. 5.2. Assess the reasonableness of a solution. 5.3. Justify the solution to a problem. 5.4. Predict implications of the solution to the given problem. 5.5. Restart the problem solving approach if the solution to a problem is incorrect.

Mapping of a Sample Assessment to Pre-Business Learning Outcomes

Sample Assessment: Identify 3 credit cards that can be used to purchase the same item, yet have different interest rates. Compare their interest rate policies for each credit card. If there was $1000 owed on each credit card, how much interest would accumulate in one year for each card? What is the minimum monthly payment? How long would it take to pay off the credit card if the minimum amount was paid? What would be the minimum monthly payment if this amount was paid off in one year?

LEARNING OUTCOMES	SAMPLE ASSESSMENT MAPPING
1. At the end of this course, the student will be able to analyze information from business-based mathematics problems to identify a problem solving strategy.	Identify that this scenario is a compound interest problem. Note that numbers calculated in business problems are taken to 5 decimal places, while money is rounded to 2 decimal places. Note that a comparison will be performed involving a 1 year interest period for 5 years.
2. At the end of this course, the student will be able to select the correct formula(s), and computational tools and strategies to solve basic business-based problems.	Identify a compound interest formula and note the variables and their role in the formula. Decide whether the calculations will be performed using a calculator (on-line or hand-held) or a spreadsheet program. Have available a calculator suggested by the program. Review how to use an Excel spreadsheet application. Consider overall that there are 5 questions in the problem: (1) a comparison of interest rates of 3 cards; (2) identification of the interest accumulated over one year for each credit card; (3) identification of the minimum payment and how it changes monthly as the balance is reduced; (4) calculation to a $0 balance for each card; and, (5) calculation of the amount with a 12 month payment plan. Create a spreadsheet that contains a page for the solution for each question.
3. At the end of this course, the student will be able to apply an operation, method, or strategy to solve a business problem.	Substitute any known values into the formulas regardless of the approach used. Solve the formulas using principles of algebra. Create an Excel spreadsheet to solve each question in the problem.
4. At the end of this course, the student will be able to model real world business problems using linear relationships.	Input all known values into the formulas and calculate the unknown values. Use the principles of algebra to rearrange and solve the formulas for any unknown variables. Use rounding rules to determine answers within 2 decimal places.

5. At the end of the course, the student will be able to justify the solution to a business problem to ensure its accuracy and validity.	Draw conclusions from each solution and consider the reasonableness of the answers. Consider the answers in terms of each other, e.g., a declining balance should be reflected in numbers lower than began with. Relate this solution to someone you may know who has a credit card balance. While some credit cards may have a higher interest, identify and discuss the benefits of each card to the cardholder as a summary of results.

Business Diploma

LEARNING OUTCOME	LEARNING OBJECTIVES
1. At the end of this course, the student will be able to utilize a problem solving approach to solve business mathematics problems such as, simple interest, compound interest, loans, mortgages, annuities, perpetuities, and investment planning and analysis.	1.1. Identify and define terms that describe business concepts. 1.2. Identify formulas, computational tools, and problem solving strategies needed to plan a solution to business mathematics problems. 1.3. Calculate the solution. 1.4. Check results and interpret the answer in terms of the given problem.

LEARNING OUTCOME	LEARNING OBJECTIVES
2. At the end of this course, the student will be able to determine the type of business problem by analyzing terminology.	2.1. Develop a glossary of terms related to business concepts such a, simple interest and compound interest. 2.2. Describe the relationship of merchandising concepts to invoicing and profit. 2.3. Describe the relationship between break-even point and profit and loss. 2.4. Differentiate between simple and compound interest. 2.5. Differentiate between mortgages and loans. 2.6. Differentiate between several types of annuities. 2.7. Differentiate between several types of perpetuities. 2.8. Differentiate between bonds and sinking funds.

LEARNING OUTCOME	LEARNING OBJECTIVES
3. At the end of this course, the student will be able to select a problem solving strategy, formula(s), and computational tools to solve business problems.	3.1. Determine a problem solving strategy. 3.2. Select the correct formula for the business problem. 3.3. Justify how a formula is connected to a specific type of business concept.

	3.4. Select technology-based or non-technology-based computational tools as needed to explore, display, manipulate, and present data in a variety of ways (e.g., business calculators, spreadsheets, timelines, schedules, graphs).
LEARNING OUTCOME	**LEARNING OBJECTIVES**
4. At the end of this course, the student will be able to apply formula(s), computational tools, and problem solving strategies to solve business problems.	4.1. Apply the principles of algebra to solve business formulas. 4.2. Apply computational tools such as estimation, technology (calculator and software), timelines, schedules, and graphs. 4.3. Apply selected problem solving strategies.
LEARNING OUTCOME	**LEARNING OBJECTIVES**
5. At the end of the course, the student will interpret the solution in the context of the problem.	5.1. Review steps of the solution for accuracy, attempting a different strategy if incorrect. 5.2. Assess the reasonableness of the solution. 5.3. Justify the solution to the problem. 5.4. Predict the implications of the solution to the given problem.

Mapping of a Sample Assessment to Business Diploma Learning Outcomes

Sample Assessment: Many educational institutions provide scholarships to students to offset the cost of tuition, ancillary fees and other educational expenses. Investigate two different scholarships provided by your college, including one that began by an individual. Identify how the scholarship began, and what it takes to initiate and maintain the disbursements of the scholarships. Provide all calculations and a discussion of the scholarship funds.

LEARNING OUTCOMES	SAMPLE ASSESSMENT MAPPING
1. At the end of this course, the student will be able to utilize a problem solving approach to solve business mathematics problems such as, simple interest, compound interest, loans, mortgages, annuities, perpetuities, and investment planning and analysis.	Consider a problem solving approach to solve this problem: select 2 scholarship programs, gather information about the programs, identify the correct formulas used to sustain the scholarship, perform the calculations, and complete the report summary based on the results. Consider the use of a spreadsheet to perform the calculations and present the information.
2. At the end of this course, the student will be able to determine the type of business problem by analyzing terminology.	Consider that many scholarship programs are based on a type of annuity. Identify on which type of annuity the scholarship is based: ordinary simple annuities, ordinary general annuities, simple annuities due, and/or general annuities due.
3. At the end of this course, the student will be able to select a problem solving strategy, formula(s), and computational tools to solve business problems.	Substitute any known values into the formulas. Solve the formulas using principles of algebra. Perform the necessary calculations using a calculator suggested by the program or a spreadsheet program.
4. At the end of this course, the student will be able to apply formula(s), computational tools, and problem solving strategies to solve business problems.	Input all known values into the formulas and calculate the unknown values. Use the principles of algebra to rearrange and solve the formulas for any unknown variables. Use rounding rules to determine answers within 2 decimal places.
5. At the end of the course, the student will interpret the solution in the context of the problem.	Summarize the information gathered regarding the scholarships. Include how the scholarship was begun, how it is sustained, who the scholarship is intended for, and how it is distributed. Include all calculations in your answer. Check all calculations and final answers to ensure they are reasonable within the context of the problem and with respect to each other.

Chapter 5: Recommendations and Conclusions

The LODP team developed learning outcomes and supporting documents for pre-technology, pre-business, and first-year business diploma-level mathematics courses. The design modelled a framework paralleling the TCU program standards in order to make explicit the mathematics students require to be successful in their chosen career path.

In general, based on a review of the college course outlines used to generate the topic lists for the course outlines, colleges may want to review their topic lists vis-à-vis what other colleges include or do not include in their course outlines as a measure of a typical course. Discussions centring on the learning outcomes have revealed a number of additional recommendations emanating from this project. All recommendations suggest that the project continue beyond the writing and distribution of the learning outcomes. And, while it is significant that colleges give these learning outcomes personal, local attention, action is needed to move the project forward on a number of fronts. These include:

- Colleges adopt the learning outcomes in order to support a common curriculum for technology and business foundation, and first-year business diploma-level mathematics courses. This adoption could support meaningful teaching practices and resources between faculty within and among colleges. It would also ease transferability of credits for students. It would provide the framework needed for online foundation mathematics courses that could be made available to all students in Ontario. Students could take the foundation mathematics course while either at work or school in order to prepare for a college program requiring strong mathematics skills.

- Considering the disparity in mathematics topics pre-technology and pre-business mathematics courses, colleges need to reflect on topics that should be considered foundational and relevant to the discipline.

- Colleges with support from the TCU collaborate to conduct a similar learning outcomes development project for the language component of the CSAP and other mathematics courses; e.g., general arts, health sciences, technology.

- Colleges utilize the project results as a platform to respond to the upcoming EDU mathematics curriculum review.

- Colleges in collaboration with the TCU host forums to promote:
    - the adoption of the learning outcomes;
    - teaching strategies and resources that align with the learning outcomes; and,
    - the methodology used to develop the learning outcomes.

- Colleges in collaboration with the TCU develop an assessment rubric based on the learning outcomes similar to a model used in the secondary school curricular documents.
- Analyzing the learning outcomes vis-à-vis the test items generated by the ADP. It was suggested in the *Feasibility Study* that the ADP and LODP are distinct projects, yet related, "since the learning outcomes for the foundational mathematics courses will also be the learning outcomes on which the assessment will be based.[21]" Early in the development of the projects, it was

[21] Orpwood et al. (2013). Op cit. p. 44.

ascertained that there was not necessarily a direct mapping of the test items generated by the ADP to the learning outcomes, learning objectives or the topics developed by the LODP. However, the work of ADP team was still in progress at the time of this writing. It is recommended that a review be conducted examining the linkages between the two projects is warranted to ascertain whether connections exist or not.

- Publishing companies collaborate with college faculty to develop resources that reflect the curricular and pedagogical focus of the learning outcomes. Resources tailored to a common set of learning outcomes could be more economical for students and provide consistency in teaching practices.

Conclusions

Provincial forum discussions cited in the CMP 2011 *Final Report* identified three types of reforms: structural reform, which focuses on the organization of instructional time and content; curricular reform, which focuses on refining content; and, pedagogical reform, which focuses on changing teaching practice[22]. Addressing these three types of reforms is critical to student success. The LODP anticipates that cross-panel conversations in the secondary school and college education sectors will continue to speak to all three reforms. Adopting common learning outcomes have the potential to change teaching practice through the implementation of the problem solving approach and consideration of the impact the mathematical processes on teaching and learning.

In these ways and others, completion of the learning outcomes project represents a significant step towards supporting curricular and pedagogical change in Ontario's mathematics colleges. These sets of learning outcomes represent minimum common standards for first semester mathematics courses, and colleges could adopt them to meet their particular needs. The LODP recognizes the autonomous nature of colleges and their ability to make curricular decisions independent of one another. However, it is believed that the college community is ready for change in mathematics curriculum and pedagogy. To embrace this change through consideration of common learning outcomes would be a positive step towards supporting a smooth transition in mathematics learning for college students.

[22] Orpwood et al. (2011). Op cit. p. 61.

Appendix A: Pre-Technology Learning Outcomes: College Topic Mapping

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
1. At the end of this course, the student will be able to mathematically analyze technology-based problems in order to select an operation, method, or strategy.	1.1. Interpret numbers given in a problem.	1.1.1. Natural numbers, whole numbers, integers, rational numbers 1.1.2. Place value of integers 1.1.3. Representing numbers in standard and expanded notation 1.1.4. Rounding and estimating 1.1.5. Irrational numbers 1.1.6. The number line 1.1.7. Prime and composite numbers 1.1.8. Least or lowest common multiple (LCM) & finding greatest common factor (GCF) – using factorization; rules of divisibility 1.1.9. Definition of a fraction as the quotient of 2 integers; rational numbers 1.1.10. Definitions: proper, improper, and mixed fractions 1.1.11. Equivalent fractions 1.1.12. Simplest form of a fraction: factors, multiples; divisibility rules; prime numbers; prime factorization; lowest common denominator (LCD) 1.1.13. Rational and irrational numbers
	1.2. Represent mathematical relationships using (a) diagram(s) and/or sketch(es).	1.2.1. Basic geometric concepts and properties of straight lines, angles, triangles, polygons, and solids. 1.2.2. Properties of similar triangles. 1.2.3. The Pythagorean theorem.
	1.3. Apply the Imperial and metric (SI) systems to describe measured dimensions, such as length, area, and volume.	1.3.1. Metric system of units of measure 1.3.2. Units of mass, length, area, volume, computer memory capacity 1.3.3. Metric prefixes

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
		1.3.4. Converting from one unit of measure to another: length, weight, area, volume 1.3.5. Imperial units
	1.4. Predict a reasonable answer to the problem, including the units of measure.	
	1.5. Connect technology-based problems to real-world contexts.	
2. At the end of this course, the student will be able to select an operation, method, or strategy to solve problems where there is a known procedure, algorithm, or method of solution.	2.1. Evaluate the use of proportions as a strategy for solving problems involving relationships between ratios.	2.1.1. Definition of percent as an equivalent decimal with a denominator of 100 2.1.2. Definition of percent as an equivalent fraction with a denominator of 100 2.1.3. Converting fractions or mixed numbers to percentages 2.1.4. Converting percents to fractions 2.1.5. Converting percents to decimals 2.1.6. Converting decimals to percents 2.1.7. Equivalent rates 2.1.8. Order of a ratio 2.1.9. Comparing two quantities using rates; similar units; different units (e.g., km/h) 2.1.10. Definition of a proportion
	2.2. Select the algebraic equation(s) needed to solve problems.	
	2.3. Determine if a function and its graph can be used to interpret and solve problems.	2.3.1. Definition of a function 2.3.2. Graphing a function 2.3.3. Using a function to determine dependent values

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
	2.4. Determine whether geometric and trigonometric properties and laws can be used as a strategy to solve problems.	2.4.1. Lines – parallel, perpendicular, intersecting 2.4.2. Triangles – acute, oblique, and right; area, perimeter; Pythagorean theorem, triangle sum theorem, Hero's formula; the Sine Law; the Cosine Law 2.4.3. Quadrilaterals – types; area, perimeter 2.4.4. Defining the trigonometric functions and inverse trigonometric functions in Euclidean space 2.4.5. Defining the trigonometric functions and inverse trigonometric functions on the Cartesian plane - angles in standard position 2.4.6. Applying trigonometry to solving problems involving right triangles
3. At the end of this course, the student will be able to model technology-based problems using linear relationships, and geometric and trigonometric representations.	3.1. Analyze data using linear relationships and their graphs to predict and solve for unknown values.	3.1.1. The Cartesian plane 3.1.2. Plotting a point 3.1.3. Graphing linear equations using table of values, slope, x- and y-intercepts 3.1.4. Determining the equation of a line
	3.2. Apply geometric and trigonometric formulas and properties to design models representing problems.	

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
4. At the end of this course, the student will be able to apply an operation, method, or problem solving strategy to solve or verify the solution to a technology-based problem.	4.1. Select an operation, method, or problem solving strategy.	
	4.2. Apply the relationships between decimals, fractions, ratios, and percentages.	4.2.1. Terminology: addends, sums, differences, products, quotients, etc. 4.2.2. Properties of addition and multiplication: commutative, associative, distributive, and the identity (additive and multiplicative) properties; multiplying and dividing by “0” and “1” 4.2.3. Arithmetic operations with negative numbers 4.2.4. Order of operations: parentheses, brackets & braces 4.2.5. Addition and subtraction of fractions 4.2.6. Multiplication and division of fractions; reciprocals; division by “1” and “0;” the sign of a fraction 4.2.7. Complex fractions and order of operations
	4.3. Solve and predict unknown values using proportional relationships.	
	4.4. Demonstrate the skill to quickly and accurately perform basic mathematical operations, e.g., adding, subtracting, multiplying, dividing, and simplifying basic integer, rational and radical numerical and algebraic expressions, without the aid of technology.	
	4.5. Solve linear equations in one variable using principles of algebra involving positive and negative integers.	4.5.1. Terminology: constant, variable, monomials, binomials, polynomials, terms, like terms, expression, equation 4.5.2. Standard and general forms of an equation 4.5.3. Solving for one variable in an equation 4.5.4. Substituting for a variable

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
	4.6. Manipulate equations and formulas used in technology-based problems using the properties of exponents and radicals.	4.6.1. Degree of terms 4.6.2. Factoring and the greatest common factor 4.6.3. Using the distributive property to simplify expressions 4.6.4. Degree of equations 4.6.5. Solving formulas 4.6.6. Definition of exponents as repeated multiplication 4.6.7. Integer exponents: the “0” exponent; the “1” exponent; negative exponents 4.6.8. Definition of a radical, square, cube, and other roots of numbers; perfect squares, perfect cubes, etc. 4.6.9. Perfect roots; roots of perfect squares 4.6.10. Order of operations - evaluating expressions with mixed operations; evaluating with parentheses, brackets & braces; more than one bracket 4.6.11. Laws of exponents 4.6.12. Calculations with expressions and equations containing exponents and radicals
	4.7. Solve problems involving two and three dimensional figures using geometric and trigonometric formulas and properties.	
5. At the end of this course, the student will be able to justify the solution to a technology-based problem to ensure its accuracy and validity.	5.1. Review the solution to a problem for reasonableness and accuracy. 5.2. Test the solution to a problem by inputting the found value into the original algorithm. 5.3. Express the solution according to the numerical expectations and context of the problem, including the correct units of measure.	

Appendix B: Pre-Business Learning Outcomes: College Topic Mapping

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
1. At the end of this course, the student will be able to analyze information from business-based mathematics problems to identify a problem solving strategy.	1.1. Identify and define terminology needed to solve a business-based mathematics problem.	1.1.1. Natural numbers, whole numbers, integers, 1.1.2. Place value of integers 1.1.3. Representing numbers in standard and expanded notation 1.1.4. Rounding and estimating 1.1.5. The number line 1.1.6. Prime and composite numbers 1.1.7. Least or lowest common multiple (LCM) & finding greatest common factor (GCF) – using factorization; rules of divisibility 1.1.8. Definition of a fraction as the quotient of 2 integers; rational numbers 1.1.9. Irrational numbers 1.1.10. Definitions: proper, improper, and mixed fractions 1.1.11. Equivalent fractions 1.1.12. Simplest form of a fraction: factors, multiples; divisibility rules; prime factorization; lowest common denominator (LCD)
	1.2. Identify units of measure and conversions found in the problem.	
	1.3. Put the mathematics into a context that reflects a business formula, equation, or procedure.	

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
2. At the end of this course, the student will be able to select the correct formula(s), and computational tools and strategies to solve basic business problems.	2.1. Determine a problem solving strategy. 2.2. Determine the formula, equation, or procedure needed to solve a business problem. 2.3. Explain how a formula is connected to the solution of a business problem. 2.4. Select technology-based or non-technology-based computational tools as needed to explore, display, manipulate, and present data in a variety of ways; e.g., business calculators, spreadsheets, timelines, schedules, graphs, and on-line calculators such as mortgage calculators and student loan repayment loan calculators from national student loan websites, and currency calculators.	
3. At the end of this course, the student will be able to apply an operation, method, or strategy to solve a business problem.	3.1. Demonstrate the skill to quickly and accurately perform basic mathematical operations, e.g., adding, subtracting, multiplying, dividing, and simplifying basic integer, rational and radical numerical and algebraic expressions, without the aid of technology.	3.1.1. Terminology: addends, sums, differences, products, quotients, etc. 3.1.2. Properties of addition and multiplication: commutative, associative, distributive, and the identity (additive and multiplicative) properties; multiplying and dividing by “0” and “1” 3.1.3. Operations with negative numbers 3.1.4. Order of operations: parentheses, brackets & braces 3.1.5. Rounding and estimating answers
	3.2. Express numbers in the form of fractions, decimals, and per cents to solve problems related to ratio and proportion.	3.2.1. Definition of percent as an equivalent decimal with a denominator of 100 3.2.2. Definition of percent as an equivalent fraction with a denominator of 100 3.2.3. Converting between fractions, including mixed numbers, percents, and decimals 3.2.4. Equivalent rates 3.2.5. Order of a ratio

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
		3.2.6. Comparing two quantities using rates; similar units; different units (e.g., km/h) 3.2.7. Definition of a proportion
	3.3. Solve equations using fundamental principles of arithmetic involving positive and negative integers	
	3.4. Solve business-based formulas using the properties of exponents and radicals.	3.4.1. Definition of exponents as repeated multiplication 3.4.2. Integer exponents: the “0” exponent; the “1” exponent; negative exponents 3.4.3. Expanding numbers 3.4.4. Definition of a radical, square, cube, and other roots of numbers; perfect squares, perfect cubes, etc. 3.4.5. Perfect roots, roots of perfect squares
	3.5. Solve single variable algebraic equations in real world business problems using the principles of algebra.	3.5.1. Terminology: constant, variable, monomials, binomials, polynomials, terms, like terms 3.5.2. Degree of terms 3.5.3. Terminology: expression, equation 3.5.4. Factoring and the greatest common factor 3.5.5. Using the distributive property to simplify expressions 3.5.6. Degree of equations 3.5.7. Standard and general forms of an equation 3.5.8. Solving for one variable in an equation 3.5.9. Substituting for a variable 3.5.10. Solving formulas

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
4. Model real world business problems using linear relationships.	4.1. Plot points in the four quadrants of the Cartesian plane using integral ordered pairs.	4.1.1. Sign convention & quadrants of the Cartesian plane
		4.1.2. Plotting a point in all 4 quadrants; connecting 2 points; definition of slope
	4.2. Represent business relationships with bar graphs and circle graphs.	4.2.1. Bar graphs, circle graphs, pictographs
	4.3. Represent linear relationships in a business analysis context with tables of values, slope-intercept form, standard form, and x- and y-intercept form.	4.3.1. Linear equation in slope-intercept form 4.3.2. Graphing linear equations using table of values, slope, x- and y-intercepts
5. At the end of the course, the student will be able to justify the solution to a business problem to ensure its accuracy and validity.	5.1. Review steps of the solution for accuracy, attempting a different strategy if incorrect.	
	5.2. Assess the reasonableness of a solution.	
	5.3. Justify the solution to a problem.	
	5.4. Predict implications of the solution to the given problem.	.
	5.5. Restart the problem solving process if the solution to a problem is incorrect.	

Appendix C: Business Diploma Learning Outcomes: College Topic Mapping

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
1. At the end of this course, the student will be able to utilize a problem solving approach to solve business mathematics problems such as, simple interest, compound interest, loans, mortgages, annuities, perpetuities, and investment planning and analysis.	1.1. Identify and define terms that describe business concepts.	1.1.1. Read a business mathematics problem and identify a specific business concept. 1.1.2. Identify the known information and known variables.
	1.2. Identify formulas, computational tools, and problem solving strategies needed to plan a solution to business mathematics problems.	1.2.1. List business-related formulas. 1.2.2. Know how to use a business calculator 1.2.3. Problem solving strategies: guess and check, look for a pattern, make a systematic list, make and use a drawing or model, eliminate possibilities, work backwards, simplify the original problem, develop alternative original approaches and analyze keywords.
	1.3. Calculate the solution.	1.3.1. Solve the equation with or without using a business calculator.
	1.4. Check results and interpret the answer in terms of the given problem.	1.4.1. Check the reasonableness of the answer calculated or estimated 1.4.2. Write a statement with the given answer in the context of the problem.

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
2. At the end of this course, the student will be able to determine the type of business problem by analyzing terminology.	2.1. Develop a glossary of terms related to business concepts such a, simple interest and compound interest.	2.1.1. Terms: simple interest, compound interest .
	2.2. Describe the relationship of merchandising concepts to invoicing and profit.	2.2.1. Terms: single trade discounts, series of trade discounts, equivalent discounts, cash discounts, partial payments, mark up, and mark down and break-even price 2.2.2. Purchasing and paying for inventory by manufacturers, distributors, and retailers 2.2.3. Terms used in invoicing 2.2.4. Timelines illustrating past, present, future and partial payments of invoicing 2.2.5. Mark up on cost and selling price and a mark down on the selling price of a product examples
	2.3. Describe the relationship between break-even point and profit and loss.	2.3.1. Terms: break-even point, net income, total costs, fixed costs, variable costs, total revenue, and profit and loss
	2.4. Differentiate between simple and compound interest.	2.4.1. Differences between future value and present value of simple and compound interest 2.4.2. Variables used in compound interest and simple interest formulas 2.4.3. Timelines illustrating the difference between future and present value 2.4.4. Equivalent payments and simple and compound interest payments 2.4.5. Timelines illustrating the relationship between equivalent payments and simple and compound interest payments

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
	2. 5. Differentiate between mortgages and loans.	2.5.1. Types of mortgages, the relationship of amortization and amortization schedules
	2.6. Differentiate between several types of annuities.	2.6.1. Terms: annuities, ordinary simple annuities, ordinary general annuities, simple annuities due, and general annuities due 2.6.2. Variables used in formulas for ordinary simple annuities, ordinary general annuities, and simple annuities due and general annuities due 2.6.3. Timelines illustrating differences between present and future value of ordinary annuities and annuities due 2.6.4.Timelines illustrating deferred annuities
	2.7. Differentiate between several types of perpetuities.	2.7.1. Terms: present value of ordinary perpetuities and perpetuities due 2.7.2. Identify the term of perpetuity 2.7.3. Timelines illustrating present value and other variables of a perpetuity and a deferred perpetuity
	2.8. Differentiate between bonds and sinking funds.	2.8.1. Term: bond 2.8.2. Term: sinking fund 2.8.3. Purchase price of a bond on the interest date and between interest dates 2.8.4. Terms: book value of a sinking fund

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
3. At the end of this course, the student will be able to select a problem solving strategy, formula(s), and computational tools to solve business problems.	3.1. Determine a problem solving strategy.	
	3.2. Select the correct formula for the business problem.	3.2.1. Formulas: simple interest: present and future value, compound interest: present and future value, annuities; present and future value of ordinary simple annuities, ordinary general annuities, and general annuities due and simple annuities due, perpetuities: present value of an ordinary perpetuity, and of a perpetuity due
	3.3. Justify how a formula is connected to a specific type of business concept.	
	3.4. Select technology-based or non-technology-based computational tools as needed to explore, display, manipulate and present data in a variety of ways (e.g., business calculators, spreadsheets, timelines, schedules, graphs).	3.4.1. Timelines illustrating variables used when calculating past, present, and future values of mortgages, loans, annuities, perpetuities and deferred annuities and perpetuities
4. At the end of this course, the student will be able to apply formula(s), computational tools, and problem solving techniques to solve business problems.	4.1. Apply the principles of algebra to solve business formulas.	4.1.1. Identifying, defining, and labeling each variable in the formula 4.1.2. Formula rearranging to isolate each variable 4.1.3. Substituting known values into the formulas 4.1.4. Solving for the unknown variable with or without a business calculator
	4.2. Apply computational tools such as estimation, technology (calculator and software), timelines, schedules, and graphs.	
	4.3. Apply selected problem solving strategies.	

LEARNING OUTCOME	LEARNING OBJECTIVES	COLLEGE TOPIC MAPPING
5. At the end of the course the student will interpret the solution in the context of the problem.	5.1. Review steps of the solution for accuracy, attempting a different strategy if incorrect. 5.2. Assess the reasonableness of the solution. 5.3. Justify the solution to the problem. 5.4. Predict the implications of the solution to the given problem.	