CLOSING THE NUMERACY GAP

AN URGENT ASSIGNMENT FOR ONTARIO

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PREFACE

This report builds on the work of the past decade by the research team of the College Mathematics Project (CMP) and the College Student Achievement Project (CSAP) based at Seneca College, of which the authors were members. In particular, the senior author was a principal author of the final reports of both CMP and CSAP over the past several years. However, the present paper, while drawing heavily from those reports, is the responsibility of its two authors.

The CMP and CSAP were research projects undertaken on behalf of the Ontario College system and supported by the Ministry of Education and the Ministry of Training, Colleges and Universities. The CMP was initiated in 2004 out of a concern by college Deans of Technology that too many students were being unsuccessful in their chosen programs and that this was related to weakness in their mathematics achievement. Accordingly, the mandate of the CMP was:

- to analyse the mathematics achievement of first-semester college students, particularly in relation to their secondary school mathematics backgrounds; and
- to deliberate with members of both college and school communities about ways to increase student success in college mathematics.

In 2012, the CMP became the CSAP, at which time the study of language achievement was added to that of mathematics and the scope of the analysis was extended to include the second semester of college programs.

Over the years, the research team has become increasingly convinced that many incoming college students lack an adequate level of numeracy to be successful at the college level, and the development and testing (in 2014) of a numeracy assessment tool has only served to strengthen this conviction. The CMP and CSAP reports of the past three years have therefore sought to highlight this conclusion and propose suggestions for addressing it. However it has become evident to the authors of this paper that broader public and political support is needed if what we call a "numeracy gap" is to be closed. This paper is therefore addressed to the Ontario public (especially the parents of school-aged children), to leaders in industry and education, and to the Government of Ontario.

Numeracy is an essential skill for life and work in the 21st century and beyond, and the Ontario economy will not develop as it could if levels of numeracy continue to decline. We argue here that a "gap" has emerged between the numeracy needs and abilities of Ontarians, a gap that urgently requires closing. The schools have an important part to play, of course, but they cannot do the job alone. Society must demand that the government make the structural and policy changes required. We offer this paper both as an indication of possible and practical ways forward and as an appeal for action.

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ABOUT THE AUTHORS

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Emily Sandford Brown

Emily is a graduate of the University of Calgary (BEd and MEd), where the focus of her work was on impediments to the learning and teaching of mathematics, and on what mathematics teachers need to know to be effective. She taught mathematics and science at the Calgary Science School (now Connect Charter) before moving to Ontario and joining the College Student Achievement Project team, first as a researcher and then as Project Coordinator for the college mathematics assessment development project. Emily is now a professor at Sheridan College, teaching business applications of calculus and algebra at the Pilon School of Business.

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EXECUTIVE SUMMARY

Numeracy is an essential skill not only for individuals who want to participate fully in a modern technological society but also for Ontario as a whole, as increasingly high levels of numeracy are fundamental to many areas of the economy. Yet evidence has been mounting for a number of years that many Ontarians, both children and adults, are lacking basic levels of numeracy. We describe this difference – between necessary numeracy and actual numeracy – as a **numeracy gap**, a gap that needs understanding, explaining and most important of all, closing.

The paper documents a range of research reports that together paint a gloomy picture of the numeracy gap in Ontario.

- The 2013 OECD survey of adult skills shows more than half of Canadians now scoring below the level required for full participation in a modern technological society, a decline in the level of numeracy a decade ago.
- The College Student Achievement Project, using data from all 24 Ontario colleges, has found that, consistently over the past nine years, more than one-third of all students taking mathematics (over 12,000 every year) are at risk of not completing their college programs because of weakness in numeracy.
- The OECD Programme for International Student Assessment (PISA) compares the numeracy of 15 year-olds internationally; in this study, Ontario students have shown a steady decline from 2003 to 2012.
- Provincial assessments at the Primary (grade 3) and Junior (grade 6) divisions have shown steady increases in reading and writing achievement over the past five years but steady decreases in mathematics achievement over the same period.

Numeracy is related to mathematics but is not exactly the same thing. Where mathematics is abstract, numeracy is concrete. Where mathematics is about conceptual knowledge and procedural skill, numeracy is about using these to solve practical problems. Where mathematics education is about obtaining correct answers to simplified problems, acquiring numeracy is about fluency and confidence in grappling with real-world and often open-ended problems. Where the agenda of mathematics education is drawn from the canon of mathematical knowledge, the process of becoming numerate draws from the tasks and challenges of everyday life. Numeracy is, in summary, the ability and the confidence to use mathematical knowledge and skills in concrete real-world situations.

It follows that while relatively few students – those entering the fields of science, technology, engineering, and mathematics (STEM) – require an educational background in relatively advanced mathematics, all students require strong numeracy skills. Some of these students will apply these in career or occupation-specific contexts (such as business, health care, social services, or teaching) but all require them for everyday living, including personal finance, leisure activities and parenting. **Numeracy for all** is therefore key to Ontario's future and to that of its citizens.

Research cited in the paper has underscored the economic benefits of improved numeracy both for individuals and society

- A Stanford University study, using OECD data, shows that a modest increase in numeracy scores corresponds with almost 20% higher wages;.
- A Harvard University study estimates that poor mathematics skills in the United States could cost that country's economy $75 trillion over the next 80 years.
- A UK report entitled "The Fear Factor" argues that "mathematics is a social justice issue" because outdated science, entrenched attitudes and the lack of role models have systematically disadvantaged women and girls. Similar factors can account for the lower levels of numeracy among aboriginal people and members of some ethnic groups.

The most important step is one of changing public and private attitudes. Whereas lack of literacy is a matter of personal shame and embarrassment in our society, a corresponding lack of numeracy is not. Indeed, many people openly claim to be unable to do mathematics. This is not an attitude found in Canada alone; it is encountered in many western (but few Asian) societies and is one that we dismiss as the "myth of the math gene". Instead, we invite Ontario to adopt and then act on the following two principles:

- Everyone can be numerate as well as literate;
- Everyone needs to be numerate as well as literate to function fully in the 21st century.

Changing public attitudes to align with these principles is key to closing the numeracy gap. We therefore recommend a province-wide public awareness campaign. The aims of this campaign would be to promote numeracy for all, to dispel "the myth of the math gene," and to raise the numeracy expectations of parents and students, employers and employees, and educational institutions and those who teach in them.

Along with this public awareness campaign, we call on the Premier of Ontario to set up a **Provincial Roundtable on Numeracy,** to develop a comprehensive strategy for closing the numeracy gap and to advise on its implementation. While schools are central to closing the numeracy gap, they cannot be left to solve the problem alone. There are roles for the private sector, for the voluntary sector as well as for the public sector. For this reason, a roundtable with a broad range of stakeholders is an appropriate vehicle to design and oversee change.

The roundtable would address a broad range of questions, some of which have already been suggested by the paper:

- How can elementary and secondary school mathematics be refocussed to the goal of numeracy for all?
- How can elementary, secondary and postsecondary education systems (including apprenticeships and adult training) be better integrated so as to increase student success especially in regard to numeracy?
- How should the mathematics curriculum for Grades 1-12 be modified to be more supportive of numeracy for all?
- How can provincial assessment play a part in advancing numeracy?
- How can teachers be better supported by improved teacher education?
- How can parents support their own and their children's numeracy development?
- What research is required both to monitor numeracy levels and to support more effective mathematics teaching?
- How can the private sector and the voluntary sector support improved numeracy?

The paper has proposed a number of suggestions in response to these questions and we invite the roundtable to consider these as part of its deliberations. If these deliberations can be translated into real changes in public attitudes and educational policy and practice, the numeracy gap can be closed and Ontario and all its citizens will be the beneficiaries.

INTRODUCTION

When schools were established and education became compulsory in Ontario, the three Rs of "reading, 'riting, and 'rithmetic" were the central focus of teaching and learning. And for the vast majority of children who left school at the age of 14, those three Rs were sufficient for them to live and work effectively in the social and economic context of the 19th and first part of the 20th centuries. They provided the foundation for everyday living and for apprenticing in most forms of work.

In those days, the knowledge and skills needed for most jobs could be acquired literally "on the job" by anyone with the basic three Rs. Whether in farming or technical fields, business, health care, or human services, most people learned what they needed from older more experienced workers, either through formal apprenticeships (as in technical trades for example) or practice-based training programs (as in nursing or accounting) or just from following the practice of experts. However the pre-requisite for all of these forms of training was a good grounding in the three Rs and this was what was expected of elementary education. Until relatively recently, few students attended secondary school for more advanced education. Even in 1945, fewer than 50% of Ontario students went on to secondary school.[1] And only a tiny proportion went on to university, those whose careers led them towards the academic life or the so-called "learned professions." This was quite normal at that time; Ontario's higher education system was in what has been called its "elite stage," before it moved to the "mass stage" in the 1960s and 1970s, and to the "universal stage" that we have now[2].

Fast forward to the present. Times have changed, both in Ontario and around the world. Now, access to jobs of all kinds as well as to postsecondary education is based on credentials representing (ostensibly) specific levels of knowledge and skill in each of many subject areas, both theoretical and practical. These credentials include university degrees and college diplomas as well as apprenticeships and, at a minimum, a secondary school graduation diploma, without which employment of any kind – beyond low wage jobs in unskilled labour and retail – is now hard to find. The focus of basic training for work has therefore passed from the workplace and into schools, colleges and universities; the credentials required for entry into work are largely those issued by educational institutions.

The technical requirements for work in all occupations have become ever more onerous because of the pace of technological and economic change and so the expectations of universities, colleges and schools have increased accordingly. As the secondary school curriculum has been made more and more "rigorous" to meet these increased expectations, so has the curriculum of the elementary school. Elementary schools are now expected not only to teach the three Rs but also much more, with responsibilities not only for an expanded curriculum but also for engaging an informed citizenry to resolve many of society's most challenging problems: racism, sexism, religious tolerance, environmental protection, national and civic pride… the list is endless.

The curriculum has become more and more complex – for the reasons we have outlined here – with the three Rs still at its core, though now they are called "literacy" and "numeracy". But as the expectations of elementary and secondary education have increased, it would seem that we have developed a number of assumptions or myths that need to be articulated and examined from time to time. These include the following:

- That levels of both literacy and numeracy in Ontario are steadily improving;
- That society (and parents in particular) appreciate the value of both literacy and numeracy to the future of their children and the economy;
- That graduates of elementary schools are adequately literate and numerate for success in secondary school and beyond;
- That literacy and numeracy belong in the curriculum of elementary schools;
- That teachers have sufficient professional background and institutional support for teaching both literacy and numeracy.

Before addressing these myths or assumptions, a word about this paper. First, literacy is a subject that, for many reasons, has attracted much more attention than numeracy, both in the research literature as well as in policy and practice.[3] In our view, this is unfortunate and therefore, in this paper, we examine numeracy exclusively. Second, while much of the data concerning numeracy is national in scope and many of the inferences and conclusions relate to Canadians regardless of where they live, most of the policy implications relate to the education systems, which are provincial in scope. Any report that deals with solutions as well as problems must therefore focus on one province or group of provinces. For example, while other reports on this subject have been written primarily from the perspective of other provinces, this one is focussed specifically on Ontario, its school curriculum and assessment policies, and its teacher education and deployment practices. Where this analysis and the recommendations that follow apply elsewhere, readers can make their own judgment.

[1] R.D. Gidney. *From Hope to Harris: The reshaping of Ontario's schools* (Toronto: University of Toronto Press, 1999), pp. 13-14.

[2] Martin Trow. "Reflections on the Transition from Mass to Universal Higher Education." *Daedalus*, vol 99, no 1 (1970): 1-42.

[3] Another recent report on numeracy in Ontario from a postsecondary perspective is by Nick Dion, *Emphasizing Numeracy as an Essential Skill.* (Toronto: Higher Education Quality Council of Ontario, 2014).

The paper falls into two major parts – the evidence for a numeracy gap and our suggestions for closing it. But before examining the evidence, we review what is meant by "numeracy" and how it relates to (and differs from) the school subject of "mathematics". The next part of the paper then argues that (a) Ontario – along with much of the rest of Canada – has a problem of declining levels of numeracy among both children and adults; and (b) this should be of greater concern than it currently is to society, parents, the business community and the Ontario government. The final section then looks to the future and examines what changes can and, in our view, should be made both in the education system and in society more generally to enable Ontario to close its numeracy gap.

WHAT IS NUMERACY?

Numeracy – the terms "quantitative literacy" or "mathematical literacy" are also used – has been defined in a variety of ways but all of them have certain common features:

- It involves mathematical knowledge and skills;
- Its purpose is the effective functioning in work and society;
- In consequence, it specifically includes the ability to use the mathematical knowledge and skills in concrete, real-world situations.

Lynn Arthur Steen, who has for many years been a strong advocate for improved numeracy, has written:

> Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Mathematics is abstract and Platonic, offering absolute truths about relations among ideal objects. Numeracy is concrete and contextual, offering contingent solutions to problems about real situations. Whereas mathematics asks students to rise above context, quantitative literacy is anchored in the messy contexts of real life. Truly, today's students need both mathematics and numeracy.[4]

This is entirely consistent with the report of the Expert Panel on Student Success in Ontario cited earlier, which points out that:

Mathematical literacy involves more than executing procedures. It implies a knowledge base and the competence and confidence to apply this knowledge in the practical world. A mathematically literate person can estimate; interpret data; solve day-to-day problems; reason in numerical, graphical, and geometric situations; and communicate using mathematics.[5]

In reflecting on the day-to-day uses of numeracy, Wade Ellis (himself a mathematics professor) comments:

> On any given day, for any one person, quantitative literacy may include reconciling a bank statement, analysing data to support or oppose a local government proposal, estimating how to split a lunch bill, debugging a program by working from assumptions to a logical conclusion, deciding which medical treatment to pursue based on statistical evidence, building a logical court case, or understanding the risks in investing for retirement.[6]

To these we could add a broad range of tasks drawn from the occupations for which college programs prepare students, including: preparing the correct dosage for a patient's medication; making sense of a company's balance sheet; estimating the cost of rewiring a house; the list is endless. But the vast majority of these tasks require not the relatively sophisticated mathematics of Grades 11 and 12 as much as the thoughtful application of fundamental skills taught much earlier.

This is the essence of numeracy: it involves both knowledge and skill combined with the performance of a real world task. In particular, we note that numeracy is not the narrowly rule-bound or rote application of learned procedures. The frequently encountered dichotomy of knowledge and skill is particularly unhelpful here. The application of numeracy skills in context – such as is the case in all of these examples – requires both theoretical and practical knowledge. Acquiring and interpreting knowledge of the context are in themselves critically useful skills. Becoming numerate is not therefore something that can be done – once and for all – at a particular grade level. Rather, a person's numeracy is continuously developed as new contexts appear in which previously learned knowledge and skill is applied over and over again. In this respect, numeracy is very much like literacy. It is not something to be taught in Grade 5 or 6 and then left to grow on its own; it must be constantly supported and developed.

Yet most authors agree that, traditionally, school mathematics has focused more on the abstract mathematical concepts and procedures and less on the real-world and open-ended problems that call for the use of contextualised mathematical knowledge and skill. Partly this is a reflection of the mathematics and mathematical education that teachers themselves have received. Partly it is also the result of a mathematics curriculum that is structured on the basis of the traditional canon which underlies the mathematics curriculum worldwide[7]. And partly it is that numeracy is something that has been left to the mathematics teachers, much as literacy was traditionally left to the language teachers. But if numeracy is essentially a context-based rather than a discipline-based set of abilities, then it should be the responsibility of all educators wherever the use of numbers or other mathematical concepts are encountered in the context of a student's experience. Numeracy can be developed in the context of music, art and drama, through history and geography, even sometimes in the study of literature as well as through mathematics and science subjects.

Of course, some students – those going on to science and engineering fields, for example – need to be well prepared through relatively advanced courses in mathematics at secondary school (as is presently the case). For most others, however, success both at work and in life generally requires high levels of numeracy, which include basic knowledge of mathematics concepts together with the skills and habits of mind required to apply these in a wide variety of real-world situations.[8]

[4] Lynn Arthur Steen. 'Mathematics and Numeracy: Two Literacies, One Language' *The Mathematics Educator* (Journal of the Singapore Association of Mathematics Educators) 6:1 (2001) 10-16.

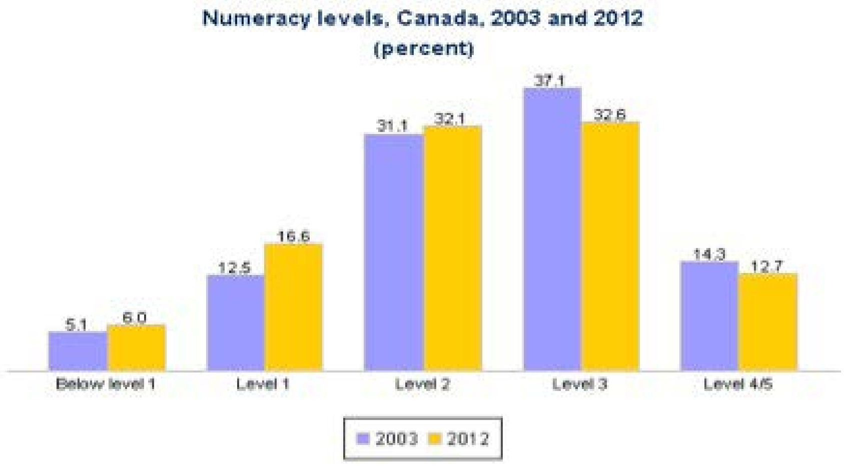
[5] *Leading Math Success: Mathematical Literacy,* Grades 7-12 (Toronto: Ministry of Education, 2004) p. 10.

[6] Wade Ellis. "Numerical Common Sense for All" p. 63

[7] Lynn Arthur Steen, Ibid.

[8] An excellent analysis of what "being numerate" might comprise has been published by the UK charity, National Numeracy, as *Essentials of Numeracy for All* (www.nationalnumeracy.org.uk/essentialsofnumeracy). The primary graphic from this analysis is reproduced (with permission) as Appendix A to this report.

EVIDENCE FOR THE NUMERACY GAP IN ONTARIO



Source: Statistics Canada, Employment and Social Development Canada, and Council of Ministers of Education, Canada. Skills in Canada: First Results from the Programme for the International Assessment of Adult Competencies (PIAAC), Table B.4.1 Literacy and numeracy - Averages and proficiency levels of population aged 16 to 65 in ALL and PIAAC, Canada, 2003 and 2012, Catalogue no. 89-555-X, Ottawa, 2013.

Figure 1. Adult Numeracy in Canada: 2003 – 2012

Several independent studies of numeracy levels (or mathematics achievement) of Canadians have appeared in the past few years. Two such studies are international in scope and sponsored by the Organisation for Economic Cooperation and Development (OECD). A third, which focuses on Ontario schools, is the annual reports of reading, writing and mathematics from the provincial Education Quality and Accountability Office (EQAO). And a fourth is the set of studies of the College Mathematics Project (CMP), now the College Student Achievement Project (CSAP), into the mathematics achievement of incoming college students. Sadly they all point to a steady decline over time in the levels of numeracy across the country. We start by looking at the numeracy levels of Canadian adults

OECD Survey of Adult Skills

The OECD Programme for the International Assessment of Adult Competencies (PIAAC) analyses the competencies of 16-65 year-olds in 24 member countries in three principal areas: literacy, numeracy, and problem-solving in technology-rich environments[9]. In the third of these categories, Canada scored higher than the average across the OECD countries. In literacy, Canada's score was not (statistically) significantly different from the OECD average. However, in numeracy – our central focus here – Canadian adults achieved significantly below the OECD average, with over 54% of Canadian adults scoring below level 3 (the median level, usually regarded as appropriate for full participation in a modern technological society).
In a similar OECD study a decade ago, 47% of Canadian adults scored below level 3 in numeracy[10]. Figure 1 shows that the Canadian numeracy gap is not just between where we are today and where we need to be but also between where we were a decade ago and where we are today.

Among the youngest adults assessed (16-24 year-olds), Canadians, while close to the OECD average, placed 16th out of 24 countries, and were outperformed by Japan, Korea, the Scandinavian countries, Germany, and the Czech and Slovak republics. Individual provinces did not differ from the national average to a statistically significant extent. The OECD data also showed that Canadian teachers fall exactly in the middle of the distribution of all postsecondary graduates[11]. Canada stands 19th out of 24 countries in the numeracy scores of both postsecondary graduates and teachers.

Once again, our weakest link appears to be numeracy. John Manley, President and CEO of the Canadian Council of Chief Executives, in a speech on the skills challenges facing the country, described these results as "a wake-up call for Canada"[12]. He asked "why – given the supposed excellence of our public education system – Canada is performing below average in comparison with many of the world's other advanced economies," adding, in response to his own question, "surely Canadians have a right to expect something better – much better – than 'below average'."

[9] Organisation for Economic Cooperation and Development (OECD). *OECD Skills Outlook, 2013: First Results from the Survey of Adult Skills.* (Paris: OECD, 2013).

[10] Organisation for Economic Cooperation and Development (OECD). *Learning a Living: First Results of the Adult Literacy and Life Skills Survey.* (Paris: OECD, 2005).

[11] A. Schleicher, "What teachers know and how that compares with college graduates around the world." *OECD Education Today: Global Perspectives on Education.* (http://oecdeducationtoday.blogspot.co.uk/)

[12] John Manley. Notes for remarks to the Canadian Club of Toronto, November 28, 2013 (downloaded from http://www.ceocouncil.ca/publication-type/speeches).

College Student Achievement Project

Over the past seven years, a research team based at Seneca College[13] working on (first) the College Mathematics Project (CMP) and (later) the College Student Achievement Project (CSAP) has been analysing the mathematics achievement of every student entering all of the 24 colleges of applied arts and technology in Ontario. Its studies have examined this achievement on the basis of demographic data (age and gender), and from the perspective of students' secondary school mathematics backgrounds (choice of courses and marks achieved).

The results of these studies have been remarkably stable over the past five years[14]. They have shown that:

- One in three students in first semester who are taking mathematics (over 12,000 students a year province-wide) are "at risk" of not completing their chosen program because of their achievement in mathematics (they get a D or F or withdraw);
- Achievement is best among mature students, especially females over 30 years of age (and who have probably not taken formal mathematics courses for many years) and worst among those who have just graduated from secondary school;
- There is a clear relationship between choice of mathematics courses in Grades 11 and 12 and achievement at college (with students who have taken College (C) oriented courses performing less well than those who have taken University (U) oriented courses) and an equally strong relationship between the marks achieved in most Grade 11 or 12 mathematics courses and subsequent achievement in college mathematics.

In addition, the CMP analysed the remedial mathematics courses that Ontario colleges have developed for those students who are in need of additional support to be successful in college diploma-level mathematics[15]. This analysis found that these courses consisted largely of topics that had been first taught, not in secondary school at all, but in Grades 6, 7 and 8. This finding has been very influential in reshaping the thinking of the CMP/CSAP research team, from its earlier focus on secondary school course choices (typically the basis of college admission requirements), and towards the levels of basic numeracy of students from a much earlier stage in their education. This new line of thought was also supported strongly by members of the secondary school mathematics community who participated in CMP and CSAP forums and attest to the low levels of numeracy of many of their students.

OECD Programme for International Student Assessment (PISA)

The OECD also conducts comparative studies of the reading, mathematics and science achievement of 15 year-olds in its member countries every three years. Each triennial study places a major focus on one of these three areas along with minor focuses on the other two. The most recent study (2012) had its major focus on mathematics and its reports are not good news for Canada nationally or for the provinces[16].

As John Richards has reported in a paper for the C.D. Howe Institute:

- From 2003 to 2012 Canada has experienced a statistically significant decline in mathematics achievement (with a scale score change of -14);
- During the same period, Ontario is one of five provinces showing a statistically significant decline in mathematics achievement (with a change of -16).

Commenting on these results, Brian Desbiens, (then) chair of the Educational Quality and Accountability Office (EQAO) noted that "Ontario students are not doing well overall, but this downward trend in math achievement adds urgency to the need to turn this story around."[17] Liz Sandals, Ontario Minister of Education, added: "we know there is still more to do when it comes to student achievement in math. That's why helping students better understand this important subject will remain our top priority for improving student achievement."[18] We note that these comments were made nearly two years ago; readers can speculate on how this sense of urgency has been perceived and acted on in schools, school boards and the province.

[13] Of which one of the authors (GO) has been a member since its inception.

[14] College Student Achievement Project. *Final Report.* (Toronto: Seneca College, 2015)

[15] Graham Orpwood et al. College Mathematics Project 2011: Final Report (Toronto: Seneca College, 2012)

[16] This section of the paper draws on a report by John Richards, *Warning signs for Canadian educators: The bad news in Canada's PISA results* (Toronto: CD Howe Institute, 2014). Richards's paper is, in turn, based on analyses from Statistics Canada and the Council of Ministers of Education, Canada.

[17] Education Quality and Accountability Office (EQAO). *News Release*, December 3, 2013.

[18] Ontario Ministry of Education, *News Release*, December 3, 2013.

Provincial Assessments of Mathematics Achievement

In Ontario, the Education Quality and Accountability Office (EQAO) is mandated to conduct annual assessments of all students in Grades 3 and 6 reading, writing, and mathematics and in Grade 9 mathematics and to report the results of these assessments publicly. Over the years, the results of these province-wide assessments have shown regular improvements in both reading and writing but more often than not declines in mathematics.

Figures 2a and 2b show the percentages of students achieving the provincial standard in mathematics in the Primary Division (Grade 3) and the Junior Division (Grade 6) over the past five years. The declines are more striking when compared with the increases in both reading and writing for both divisions. But they are hardly surprising in the light of the OECD studies cited earlier. Commenting on these results, Bruce Rodrigues, EQAO CEO, stated: "These are concerning trends that need to be reversed if we are to ensure that students are adequately prepared for future success."[19]

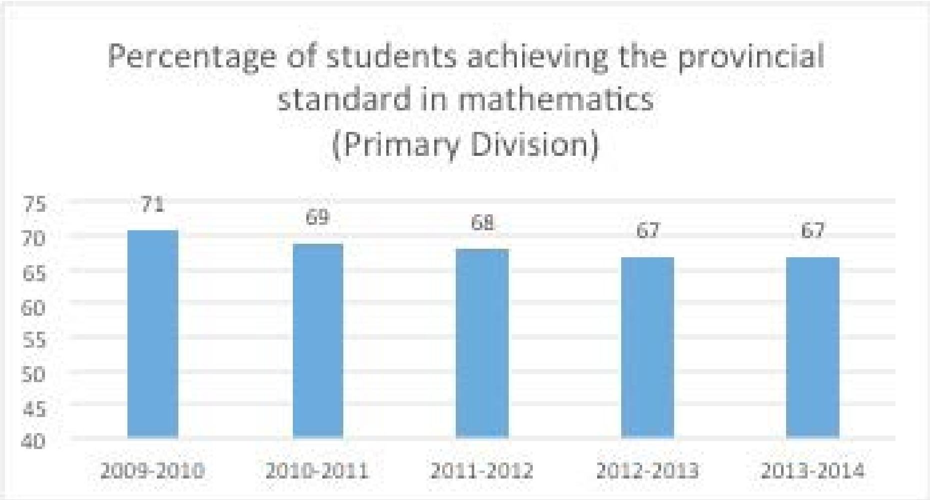


Figure 2a. Comparison of EQAO mathematics results over time: Primary Division [20]

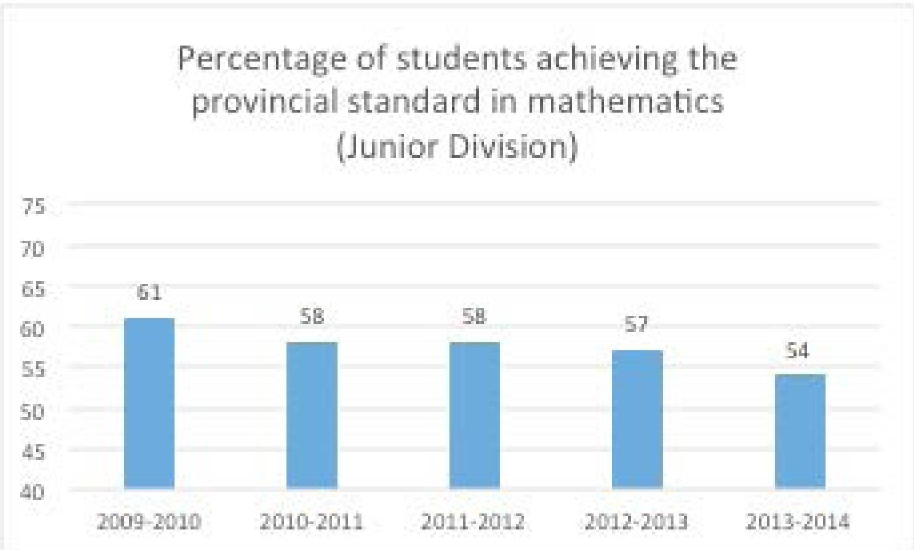


Figure 2b. Comparison of EQAO mathematics results over time: Junior Division [21]

[19] Bruce Rodrigues. "Message from the CEO" in *Ontario Student Achievement* (Toronto: EQAO, 2015).14. College Student Achievement Project. Final Report. (Toronto: Seneca College, 2015).

[20] Education Quality and Accountability Office. *Highlights of the Provincial Results 2013-2014* Primary and Junior Divisions (Toronto: EQAO, 2015).

[21] Education Quality and Accountability Office. *Highlights of the Provincial Results 2013-2014* Primary and Junior Divisions (Toronto: EQAO, 2015).

At the Grade 9 level, the results of the provincial mathematics assessment have improved very slightly over the past five years for students in both the Grade 9 academic and applied mathematics courses.[22] However at all three levels, the absolute numbers of those who fail to reach the provincial standard in mathematics represent an average of about one-third of all students in each cohort. Interestingly, it is also the proportion of students that the CMP and CSAP studies find to be "at risk" at the college level.

Summary: Why does a numeracy gap matter so much?

The challenge of numeracy in Ontario has been a major theme of the provincial forums hosted by the College Student Achievement Project (CSAP). Craig Alexander, Senior Vice President and Chief Economist of the TD Bank Group and keynote speaker at the CSAP Forum in 2013 discussed the critical importance of literacy and numeracy and the disappointing Canadian results of the OECD adult skills survey (as noted earlier). He argued that an investment in literacy and numeracy was the best possible one for governments, businesses and individuals to make. He cited an analysis of the OECD data by Stanford researcher Eric Hanushek, who found that (for Canada) a one-standard deviation increase in numeracy scores is associated with a 19.3% wage increase among prime-age workers[23]. The average return for all 24 countries is 18%.

This impact of improved numeracy levels is also supported by research from outside Canada. A recent Harvard study estimated that poor mathematics skills in the United States could cost that country's economy $75 trillion over the next 80 years[24]. Eric Spiegel, a leading American CEO pointed out recently that "over the past decade, STEM (Science, Technology, Engineering and Mathematics) job openings grew three times faster than non-STEM jobs. STEM workers are expected to earn, on average, 26 per cent more than their non-STEM counterparts." He went on to point out that his concern was not just for scientists and engineers but also for sales and marketing staff who also need a STEM background and to be "fluent in the language of medicine, energy, and high-tech manufacturing."[25] In Canada, while the numbers vary somewhat, the needs are similar as studies such as that of Rick Miner have shown[26].

The economic benefits of improved numeracy are not for business and the national and provincial economy alone; they are of direct benefit to the students themselves. Carnevale and Desrochers point out that "the value of quantitative reasoning has surged at each of the great economic divides: in the shift from agriculture to an industrial economy and most recently in the shift from an industrial to a knowledge economy."[27]

They claim that:

- "mathematical ability is the best predictor of the growing wage advantages of increased postsecondary educational attainment;
- "improvements in mathematical skills account for at least half of the growing wage premium among college-educated women;
- "although the wage premium has increased across all disciplines, it has increased primarily among those who participated in curricula with stronger mathematical content, irrespective of their occupation after graduation."[28]

These authors go on to argue for what they call the "democratization of mathematics" which, they insist, "does not mean dumbing down. It means making mathematics more accessible and responsive to the needs of all students, citizens and workers."[29]

Craig Alexander also showed that, despite its focus on equality, Canada was among those countries having both high wage inequality and high skill inequality. In other words, Canadian education is even failing to deliver on equality, one of its most cherished principles. A recent study from the UK on gender and mathematics opens with the words: "mathematics is a social justice issue."[30] Callan's comprehensive report traces the historical roots of what she describes as the "fear factor" that underpins the myth that women in particular are less able to be successful than men at mathematics and therefore less able to participate in those professional fields in which mathematics plays a foundational role.

In conclusion, we can see that:

- There is clear evidence of the existence of a real and growing gap between the numeracy abilities of Ontarians and their numeracy needs;
- This numeracy gap is impacting economic opportunities of individual citizens as well as those of the province as a whole;
- An investment in closing the numeracy gap can advance the goals of both social equity and economic prosperity.

The final part of this paper builds on these conclusions and proposes some radical (but not cost prohibitive) suggestions for creating a numerate as well as a literate society in Ontario.

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22 Education Quality and Accountability Office. *Highlights of the Provincial Results 2013-2014* Grade 9 Mathematics (Toronto: EQAO, 2015).

23 E. Hanushek et al. R*eturn on Skills around the world: Evidence from PIAAC.* OECD Working Paper #101. Paris: OECD, 2013.

24 Paul Peterson, Ludger Woessmann, Eric Hanushek, and Carlos Lastra-Anadon. *Globally Challenged: Are US Students Ready to Compete?* 2011

25 Blog by Maureen Downey. http://blogs.ajc.com/get-schooled-blog/2011/12/05/

26 Rick Miner. *People without jobs – Jobs without People: Ontario's Labour Market Future.* (Toronto: Miner Management Consultants, 2010)

27 Anthony Carnevale & Donna Desrochers. "The Democratization of Mathematics." In *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges* (Princeton, NJ: National Council on Education and the Disciplines, 2003), p. 22.

28 Anthony Carnevale & Donna Desrochers. Ibid.

29 Anthony Carnevale & Donna Desrochers, Op. cit., p 29.

30 Samantha Callan. *The Fear Factor: Maths anxiety in girls and women.* A report for Maths Action. (London: The Learning Skills Foundation, 2015).

CLOSING THE NUMERACY GAP

Solutions to the lack of numeracy in Ontario, while urgently needed, are hindered by a fundamental misconception, commonly held in western countries. Many people believe that even a basic mathematical ability is innate, that some have it and some do not. It is not uncommon to hear people say: "Oh, I could never do math" or "You may not have any math ability," using this as an excuse for their children's struggles with mathematics in school. This way of speaking implies that people believe that mathematical ability is somehow genetically transmitted and that you either "have it" or you don't. It follows that if you don't, then no amount of hard work or better teaching will make a difference. Canadians are not unique in tending towards these beliefs and attitudes. Americans, the British and many Europeans admit to sharing belief in this myth. People of Asian origins, by contrast, tend to see numeracy as something that can be achieved through hard work[31].

Interestingly, you do not hear such sentiments concerning literacy. You don't hear people excuse their children from reading or writing. Rather, it is recognised that some people need to work harder or differently in order to achieve proficiency. In fact, illiteracy is a matter of shame and embarrassment where it occurs and people who cannot read or write often take elaborate means to hide the fact from becoming public. By contrast, a lack of numeracy is often a matter for bragging, laughing off or an excuse for themselves and others.

It is high time, in our view, that we as a society reject the "myth of the math gene" and instead adopt the beliefs that:

- Everyone can be numerate as well as literate;
- Everyone needs to be numerate as well as literate to function fully in the 21st century.

Achievement of the goals implied by these beliefs will take decisive and strategic action not only by the Ministry of Education but also by the Premier of Ontario and will need to be supported by the private and voluntary sectors and by society at large. The second part of this paper addresses a range of these, starting with the most fundamental societal change and moving into more specific, even technical policy changes by the Ministry of Education and other educational partners, which together can support Ontario's move towards numeracy for all.

Public attitudes toward numeracy

If Ontarians are to be as numerate as they need to be over the next decade, the most fundamental change that is required must be in public attitudes. Without this, politicians will be unwilling to act boldly to make structural or policy reforms, educators will be less inclined to develop curricular or pedagogical changes, and most importantly, parents will not have the expectations of their children in regard to numeracy that they already have in relation to literacy.

A province-wide or, better, a national public awareness campaign to promote numeracy, to dispel the "myth of the math gene," and to raise the expectations of parents and students, employers and employees, educational institutions and those who teach in them, should be the place to start. In our view, such a campaign should be led by a national, not-for-profit organisation with a proven track record of promoting and supporting education in fields such as mathematics and science, [32] and supported financially by both governments and the private sector. The campaign could include such elements as TV information spots, advertisements in newspapers, public transport, and elsewhere, extensive use of social media, plus more focused communications addressed to students, parents and educators.

In line with that campaign, the Ministry of Education and its educational partners (school boards, schools, teachers' organisations, faculties of education, EQAO, the Ontario College of Teachers, among others) need to address those structural issues along with any policies and practices that currently impede increased numeracy. In this paper, we address the following:

- Elementary school mathematics
- Secondary school mathematics
- Assessment and remediation
- Teacher education and deployment
- Research

In each section, we identify what we consider to be the factors most likely to maintain the numeracy gap and make our own suggestions for possible changes. Readers are invited to debate these proposals, to add to them, modify them, or replace them. However, mere rejection of these proposals will not eliminate the underlying problems. Similarly, agreement with the premises of this report (outlined earlier) is not consistent with an unwillingness to implement change. We invite all who believe that the numeracy gap needs to be closed to ask themselves: "knowing what we now know, what are we going to do differently?"

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[31] Stevenson, H. W. and Stigler, J. W. The learning gap: *Why our schools are failing, and what we can learn from Japanese and Chinese education.* (New York: Summit Books, 1992).

[32] Such as *Let's Talk Science*, a national charity devoted to advocacy and support of children's learning in STEM fields. www.letstalkscience.ca.

Elementary School Mathematics

The evidence concerning declining levels of numeracy in Ontario, summarised in the first part of this report, points inevitably towards the mathematics experience of many – maybe most – children in the elementary grades of Ontario schools. Certainly, the topics found not to have been learned successfully by older students are those which first appear in Grades 1-8, as the research has shown.

Some commentators have claimed that the root of the problem is to be found in the use of what they describe as "discovery-based" or progressive approaches to teaching. We reject this analysis as simplistic. First, such claims are not based on evidence of how mathematics is actually taught in Ontario schools. Second, as Professor Jo Boaler has argued, on the basis of research in many mathematics classrooms in both the UK and the US, "such categories do not mean much, and that both camps (i.e. progressive and traditional) include effective and ineffective teaching." [33] We draw on Boaler's own research-based analysis of the sources of students' negative experiences later. But third, we find that an ideological analysis in this case – rather like the arguments over phonics and whole language in the teaching of reading – is unproductive. Experienced teachers with a good command of their subject and of ways of teaching it are inclined more towards a pragmatic rather than dogmatic approach to their classroom practice. They will use "whatever works best" in each specific situation.

We therefore turn to look at the mathematics curriculum for Grades 1-8 in current use in Ontario and to its implementation in classrooms. The Ontario mathematics curriculum is, in our view, a comprehensive document, containing much of the foundational knowledge and skills that the College Mathematics Project team found to be important for success beyond secondary school, into college and for basic numeracy for everyday living. While we do not see the fundamental numeracy problem as one that can be solved by the rewriting of the curriculum, there are a few points that should be taken into consideration when the cyclical review of the mathematics curriculum is next undertaken.

First, we find the mathematics content of the curriculum to be very dense. As a result, there is the risk that the need to 'cover' so much content leaves little instructional time for a deeper exploration of the connections among the various concepts. There is a need for the connections amongst mathematics concepts to be more explicit. The curriculum already states:

> The program in all grades is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways. To support this process, teachers will, whenever possible, integrate concepts from across the five strands and apply the mathematics to real-life situations.[34]

This is important in two ways. As students move through their mathematics journey, the ways that concepts are connected need to be well understood and experienced. Teachers who are themselves strong mathematically will therefore make connections amongst mathematics concepts so that students gain a rich understanding of the landscape that each concept occupies. Making these connections transparent for students and making "numeracy across the curriculum" a reality will help prepare students for the more complex mathematics that will come later.[35]

One specific concern raised across the country and vehemently argued in the province of Ontario is the fact that the memorization of multiplication tables is not a stated outcome in the K-8 curriculum; this is taken as indicative of a general lack of focus in the curriculum on basic essential skills. Memorizing and recalling basic facts or relevant information from memory sits at the lowest level of Bloom's Taxonomy, while the ability to use numbers and think mathematically in everyday life – the essence of being numerate – denotes learning at a higher level. Achieving higher levels of learning, however, depends on having first attained prerequisite knowledge and skills at lower levels, whether or not these are stated explicitly as curriculum outcomes. There is little doubt in our mind that acquiring a working knowledge of basic mathematical facts (including the multiplication tables) is something that should be expected of all students and should therefore be identified explicitly as learning outcomes. Otherwise, some teachers may regard their explicit absence from the curriculum as implying a lack of importance.

Our final comment on curriculum policy centres on the early adoption of technology, particularly calculators, in mathematics. This can lead to a dependence on technology that persists and grows as students go through the school system, leaving them unable or unwilling to do simple calculations (such as simple multiplication or addition) without the aid of technology. Over the past two years, the CSAP team was mandated to develop an assessment of basic mathematical or numeracy skills. During this development project, there was much discussion about whether to allow students to use a calculator when writing the assessment. While it was recognised that students are permitted the use of a calculator when writing the Grade 6 EQAO mathematics assessment, college advisors to the assessment development team were clear about their expectation that students coming into college programs must be able to perform basic mathematical calculations without the use of a calculator. Some college programs (such as nursing and emergency medical services programs, for example) do not allow calculator use at all.

[33] Jo Boaler. *The Elephant in the Classroom: Helping Children Learn and Love Maths* (London: Souvenir Press, 2009) p. 35.

[34] *The Ontario Curriculum, Grades 1-8: Mathematics* (Toronto: Ministry of Education, 2005), p. 8.

[35] This is also a major theme of the Ministry of Education's support document *Supporting Numeracy: Building a Community of Practice K-12* (Toronto: Ministry of Education, 2012)

It was therefore decided that the CSAP assessment items would be created with numbers which could be worked easily without a calculator by students who have solid numeracy skills. Despite this, the overwhelming feedback from both teachers and students indicated that students were so accustomed to having easy access to a calculator that many were simply unable to do simple calculations without one.

Ontario curriculum policy "recognizes the benefits that current technologies can bring to the learning and doing of mathematics. It therefore integrates the use of appropriate technologies, while recognizing the continuing importance of students' mastering essential arithmetic skills."[36] We note that, while calculator use is mentioned in the Ontario mathematics curriculum as early as Grade 1, in other provinces, calculators and other technology are described as "useful tools to enrich learning and feed the curiosity of young students."[37] It also states that the curriculum outcomes are to be met without technology up to and including Grade 3. The ability to think mathematically and perform simple operations without the use of a calculator sits at the very heart of numeracy. While technology has made routine calculations easier and faster, it is our view that technology should be introduced to students for use in the classroom only after a firm conceptual understanding of mathematical operations has been developed, together with the ability to apply them accurately and rapidly. Even then, technology should be used with discretion.

Despite these suggested adjustments to the "intended curriculum," it is evident that the real concerns are with the "implemented curriculum", with what actually takes place in classrooms. We strongly endorse the remarks of Liz Sandals, Minister of Education, at a January 2014 press conference, when she said in relation to the declining mathematics achievement: "Curriculum's an ongoing review and we've already checked in with the experts to make sure that the curriculum is there. It's more about making sure that math is taught really well in elementary school."[38] And, in its efforts to promote "teaching math really well," the Ministry has been active. It has developed a 2014-2015 mathematics action plan based on a previously published set of foundational principles for improvement in mathematics K-12.[39] Both the principles and the action plan are a clear step forward in the right direction. However our report suggests that more radical steps may be needed if the goal of numeracy for all is to be achieved.

In her analysis of "what's going wrong in classrooms" based on her research both in the United States and in England, Jo Boaler identifies three fundamental problems with many mathematics classrooms.[40]

- Passive learning – where teachers demonstrate methods followed by students being given sets of questions involving practice of the methods; such an approach, claims Boaler, requires mindless following of procedures rather than thinking or reasoning, and is ultimately mind-numbing and (for the students) lacking in purpose. The totality of mathematics then becomes the memorization of hundreds of such methods or rules. As one student, Kate, told her: "We knew how to do it. But we didn't know why we were doing it and we didn't know how we got around to doing it….I can get the answer, I just don't understand why."[41]
- Learning without talking – where students work on math problems alone and in silence, rather than by talking about them with their peers. Real understanding and real problem solving is strengthened by discussion with others, a process that also requires reasoning, thinking and listening.
- Learning without reality – where many of the so-called problems given to students are wholly artificial. In fact, if students bring their real-world knowledge to bear on the problems often given to them in mathematics classes, they would get the answers wrong!

We are of the belief that these characteristics of ineffective teaching identified by Boaler from the US and the UK might also be encountered in many Ontario schools. She goes on to address how these problems can be solved in practice by the use of more effective problem-solving approaches. And that is quite consistent with Ontario policy: the very first statement (in the Ontario Foundational Principles document) about effective mathematics instruction is that it is "based on problem solving and investigation of mathematical concepts." [42]

It is our view that all of this analysis and advice would make a very positive difference to Ontario mathematics teaching if the teachers had an adequate background, education and training for the task that is expected of them. Rather, we find from EQAO's survey of Grade 3 teachers that:

- *83% have no postsecondary background in mathematics;*
- *82% said that they received mathematics teaching courses as part of their preservice teacher education, but 71% have not taken an Additional Qualification course for Primary/Junior mathematics*

Grade 6 teachers had similar responses, with 80% having no postsecondary mathematics, and 69% have not taken a Primary/Junior mathematics AQ course. Ontario's teachers are, in our view, as professional and dedicated as teachers anywhere in the world. However, in common with teachers in many other jurisdictions, most Ontario teachers are not provided with the training required for effective mathematics teaching. This is a theme that we shall take up in a later section of this report (on teacher education).

[36] *The Ontario Curriculum,* op cit. p. 5

[37] https://www.wncp.ca/media/38765/ccfkto9.pdf p. 15

[38] Liz Sandals, press conference, January 8, 2014.

[39] Ministry of Education. *Paying Attention to Mathematics Education.* (Toronto: Ministry of Education, 2011); "2014-15 Mathematics Action Plan" Memorandum from George Zegarac (Deputy Minister) to Directors of Education, January 21, 2015.

[40] Jo Boaler. op. cit. pp 32-50.

[41] Boaler, op. cit. p. 37.

[42] Ministry of Education. *Paying Attention to Mathematics Education*. p. 6.

Secondary School Mathematics

It is our position that the primary goal of the school mathematics curriculum should be to make provision for all students to achieve the highest level of numeracy needed for participation in their personal, social and economic lives. That does not imply that all students should take the same mathematics courses throughout secondary school but it does challenge the structure and aims of the current secondary school mathematics curriculum in Ontario.

Secondary school teachers have reported (at CMP and CSAP forums and also as feedback in the field trials of a recent CSAP assessment project) that the secondary school mathematics curriculum is so full that they cannot revisit numeracy topics from the elementary school mathematics curriculum – topics such as fractions, ratios, and percentages – even though they recognise that many of their students need to do so. This means that key topics, essential for developing and maintaining a good level of numeracy, but which have been taught in Grades 5, 6 or 7 (for example), do not feature in the curriculum at Grades 9, 10 or 11, nor is there time to include them. In addition, the evidence from the CMP and CSAP show that while all students taking college mathematics have secondary school graduation diplomas (and thus a minimum of three secondary school mathematics credits), many of them still struggle with basic numeracy topics taught in elementary school.

In addition, there are too many mathematics courses in the mathematics curriculum. At Grades 11 and 12, Ontario has ten mathematics courses for students to choose from, where most other provinces have six. As a result, the majority secondary schools in the province (and almost all those in the north and in other less densely populated areas) cannot deliver all of these courses because of modest enrolment levels.[43] As both Alan King's research and the CMP reports[44] have shown, this disadvantage has not impacted all students equally: the system serves students headed for university relatively well but those oriented to college and the "workplace" less so. From a college perspective (and somewhat less so from a university perspective as well), this has also made the problem of defining admission requirements very challenging, as the CMP and CSAP studies have shown. It is unrealistic to "require" students to have a particular Grade 12 mathematics course (such as, for example, MCT4C, Mathematics for College Technology) if many secondary school students do not have access to that course in their local secondary school.

In our view, these problems originate in the concept of "destination" used by the Ministry of Education to differentiate secondary school courses at the senior level (Grades 11 and 12). Since the mid-1990s, the Ontario curriculum has differentiated grade 11 and 12 courses in terms of types of institution: universities, colleges, and workplace, designating them as being preparation for one or more of these institutions. However, these institutions do not have common sets of expectations for students' preparation in mathematics. For example, a university engineering program has a very different mathematics requirement from those of a university business or history program. U-designated mathematics courses are not equally appropriate for all of these. The same is true of college programs (as the CMP and CSAP research has documented) and C-designated mathematics courses are not equally appropriate here either. In addition, many colleges are now offering 4-year degree programs, rendering the C designation for secondary school courses particularly inappropriate.

We believe that the challenge for the education system is to implement a new focus in secondary schools on numeracy for all while also preparing all students in appropriate and clear ways for future postsecondary and career paths. Undertaking this while keeping the present mathematics course structure in place is clearly impossible. Numeracy is not just another course to be added into an already overfull curriculum. We have therefore begun to re-imagine the overall mathematics curriculum from Grade 9 through Grade 12 in ways that would make numeracy the central goal for all students.

We propose that the concept of "destination" should be reformulated with postsecondary programs or occupations rather than institutions in mind. In this model, one sequence of courses could be designed for students intending to specialise in STEM subjects – science, technology, engineering, and mathematics – at any postsecondary institution. A second sequence could be designed for those aiming for academic and career paths – again, at any institution – that require some use of mathematics though not at such an advanced level; examples include fields such as business, health care, education, social sciences and human services. Finally, a third sequence could be for those headed for academic and career paths that do not require any study of mathematics beyond the secondary school level – the arts and humanities, for example, whether at colleges or universities. Students not planning to continue to either college or university could take any of these sequences.

Figure 3 shows these three sets of courses in relation to secondary school grades and postsecondary program destinations (for those students continuing to postsecondary education). This approach towards a new mathematics program in secondary school is, of course, simply an outline. The details need careful thought and deliberation. For example, provision should be made for those students whose aspirations beyond secondary school change over time and who may therefore wish to switch from one stream to another.

43 A. King et al. Double Cohort Study: Phase 4. (Kingston: Social Program Evaluation Group, Queen's University, 2005).

44 L. Schollen et al. College Mathematics Project 2008: Final report (Toronto: Seneca College, 2009).

<table>
<tr><th>Grade</th><th>Mathematics for further education in STEM subjects</th><th>Mathematics/Numeracy for further education/careers</th><th>Mathematics/Numeracy for personal and social living</th></tr>
<tr><td>9</td><td colspan="3">Numeracy for all (one course)</td></tr>
<tr><td>10</td><td colspan="3">Numeracy for all (one course)</td></tr>
<tr><td>11</td><td rowspan="2">A sequence of two or more courses similar to the university-oriented mathematics courses in the present curriculum.</td><td rowspan="2">A sequence of two courses in which basic numeracy and additional topics are embedded in contexts from a variety of career paths and everyday life.</td><td rowspan="2">A sequence of two courses in which basic numeracy topics are embedded in contexts from everyday life, including finance, leisure, personal living, etc.</td></tr>
<tr><td>12</td></tr>
<tr><td>Postsecondary program destinations</td><td>STEM programs at college and university</td><td>College and university programs with some mathematics requirement</td><td>College and university programs with no mathematics requirement</td></tr>
</table>

Figure 3. Secondary school mathematics curriculum framework

Once such a new structure is adopted, the place of numeracy in the mathematics curriculum becomes much clearer. First, agreement needs to be reached concerning what numeracy for all should comprise.[45] Then, each sequence of courses can be developed using, as key resources, professionals and academics from each field to provide authentic scenarios in which mathematics knowledge and skills are used in the real-world contexts of each field of work or study. For the specialist courses, numeracy is implicit throughout. For the second sequence of courses, numeracy topics would be contextualised in the varied real world business applications, health care applications, and social science applications. In the third sequence of courses, numeracy topics would be embedded in situations encountered by people in the course of everyday and personal life, including statistics in current affairs, estimation as practised in sport or in driving, financial numeracy, among many other topics.

We are also suggesting that, at the Grades 9 and 10 level, the focus of the curriculum for all students should be on the review and application (in new contexts) of basic mathematical concepts most of which were learned in elementary school. We see no reason to propose different sequences of courses at this level such as is the case at present. Such a change will also resolve the controversy – at least in mathematics – presently associated with the so-called streaming of students into "academic" and "applied" courses[46] . The emphasis of these two years would be the consolidation of earlier learnings, remediation where necessary, and the introduction of a limited number of important new topics (such as data analysis and statistics) that would be part of all sequences of senior mathematics courses.

It is important to note that the three sequences of courses proposed for Grades 11 and 12 have no implicit hierarchy based on ability. Unlike the present curriculum and its predecessors, each of the sequences could be appropriate for all students, regardless of their prior mathematics achievement or perceived ability. Numeracy for all means exactly that. It is also intended that schools throughout the province should aim to offer all three sequences of courses regardless of their overall enrolment.

Finally, if the sorts of changes proposed here are implemented, we would encourage the Ministry of Education then to revise the Ontario Secondary School Diploma (OSSD) requirements so that secondary school graduates are required to obtain four (rather than the present three) credits in mathematics. This is already the case in language and in most other provinces. We are reluctant to propose this change until the curriculum is reformed as it is hard to see how "more of the same" would be of help to those students who presently choose not to take any further mathematics after Grade 11.

What we have proposed here is not a new mathematics curriculum as such. Rather, we have suggested a framework for thinking about and planning a new curriculum. We believe that it would serve the needs of all students better, it would meet the needs of programs at postsecondary institutions more clearly, it would be a system that could be implemented in schools of all sizes throughout the province and, most important of all, it would be a start towards closing the gap and making Ontario more numerate.

[45] We have been impressed by the analysis developed in the UK by the National Numeracy organisation and included as Appendix A to this paper (www.national-numeracy.org.uk/essentialsofnumeracy).

[46] For further background on this controversy, see: People for Education, *Choosing Courses for High School* (Toronto: People for Education, 2014).

Assessment and Remediation

If official policy documents represent the intended curriculum and classroom teaching environments the implemented curriculum, assessments provide evidence of the achieved curriculum. As such, assessment has always played an important role in teaching and learning. However, the uses to which assessments are put vary considerably, as do their value to the learner. For example, final examinations at school or university typically play an important role in certifying (in a summative way) students' having achieved the goals of the course. They usually contribute little of direct benefit, however, to students' learning. Such assessments have been described as "assessments of learning."

A quite different approach to assessment is illustrated well by the activities of the coach in a hockey game or the director of a choir. These individuals are constantly assessing the work of the groups they are responsible for and the individuals within the groups, but their primary goal is not to provide marks or certification but to support both the individuals and the group as a whole to improve their performance. In the classroom context, teachers do the same as they work informally with students as they do project work or problem solving. Assessment of this sort contributes directly to students' learning and, as such, it is described as "assessment for learning."

In the past several years, many members of the educational community have been trying to place a greater emphasis on assessment for learning and the Ontario Ministry of Education has a clear policy statement in support of this trend[47]. At the postsecondary level, there are fewer explicit policy statements but, nonetheless, a growing recognition that assessment can (but - sadly - does not always) contribute to students' learning. A prime example of this recognition was the almost unanimous support given to the CSAP mathematics assessment development project based on assessment for learning principles.

Over the past two years, a CSAP team (including the authors of this report) undertook the development and field trialling of a diagnostic assessment in basic numeracy on behalf of the Ontario college system.[48] The final product included the assessment itself plus a set of remedial modules (developed in conjunction with Vretta Inc. the technology partner in the CSAP assessment development project) so that students can:

- Take the CSAP numeracy test on line at any time using any technology platform;
- Receive immediate feedback on their achievement, including identification of their strengths and weaknesses;
- Move directly into remedial work using tested online modules.

The overall assessment was developed with the support of secondary schools and colleges in Ontario and field-trialled by over 10,000 students during 2014. The implementation of this CSAP assessment system is now being led by Humber College but the remedial modules developed for CSAP assessment project are already in use as part of Vretta's Elevate My Math program.[49]

The aim of the CSAP assessment program as well as the Elevate My Math program is to provide opportunities for Ontario students not only at college but also in secondary school and elsewhere with opportunities for self-assessment and remediation in the area of numeracy. The further development of numeracy in Ontario can only be strengthened by initiatives such as these and we would encourage educational institutions (elementary, secondary and postsecondary) as well as parents and members of the public to make full use of them as they become readily available on line.

While assessment for learning is the clear way forward to increase numeracy in Ontario, assessment for certification has an important role to play as well. In 2001, the Government of Ontario introduced the Ontario Secondary School Literacy Test (OSSLT), the passing of which became a new requirement for secondary school graduation. This move was in response to demands from employers, postsecondary institutions and others that all secondary school graduates be required to demonstrate adequate levels of reading and writing ability. Students take the OSSLT typically in Grade 10, to enable those that do not pass at their first attempt to take the test again and then, if necessary, complete a special literacy course and still be able to graduate from secondary school with their peers.

Now, in view of the continuing decline in numeracy noted earlier, we believe that graduates of secondary schools should also be required to demonstrate their numeracy skills. We therefore call on the Ministry of Education to request EQAO to create an Ontario Secondary School Numeracy Test (OSSNT) and to revise the secondary school graduation requirements to include numeracy along with literacy. If the cost of this new assessment is a barrier to its implementation, we would suggest that the proposed numeracy test would be of more value to students, schools and the public than the current Grade 9 mathematics assessment which could be cancelled.[50]

The impact of introducing a numeracy test in parallel with the literacy test would be significant and widespread. It would be a very clear signal that the Government was serious about numeracy to students, but also to parents, educators, and society at large. This would be the single most public policy in support of numeracy for all. It would demonstrate that numeracy was not just something the Government was asking others to take responsibility for but that it was prepared to make an important policy commitment itself. It would also support the curriculum and teacher education reforms proposed elsewhere in this report.

47 Ministry of Education. *Growing Success: Assessment Evaluation and Reporting in Ontario Schools.* (Toronto: Ministry of Education, 2010).

48 Graham Orpwood and Emily Brown. *Assessment Development Project: Final Report* (Toronto: Seneca College, 2015). http://csap.senecacollege.ca/en/publications.

49 www.elevatemymath.com

50 The rationale for this is as follows. Firstly, the Grade 9 mathematics assessment only covers the Grade 9 curriculum, not that of the full Grades 7, 8 and 9 division. Secondly, its results have traditionally been associated with the secondary schools that the students have attended for a few months only; they have not been used as feedback to the elementary schools from which the students have come. Thus, unlike the assessments at the Primary and Junior Divisions, whose results have provided very useful feedback to schools, the Grade 9 mathematics results have, in our view, been under-utilized.

Teacher Education and Deployment

Our earlier recommendations on mathematics curriculum and instruction cannot be implemented without corresponding changes to teacher education. The goal of numeracy for all cannot simply be achieved by a grand announcement and expecting schools to respond. It cannot even be achieved by providing funding to teachers' organisations to support teachers who wish to take so-called Additional Qualification courses[51]. These strategies are, in our view, formulas for "more of the same" and will make little contribution to the major change implied by the "numeracy for all" mandate. Indeed, as far as we can tell, there has been little attempt made to determine the impact of the special funding.

In the present section, we address education and training for teachers in elementary and secondary schools and therein lies a problem. While both the curriculum and the majority of schools in the province are organised in terms of "elementary" (Grades 1-8) and "secondary" (Grades 9-12), teacher qualifications and teacher education programs are not. Rather, they are defined in terms of four "divisions" – Primary (Grades 1-3), Junior (Grades 4-6), Intermediate (Grades 7-10) and Senior (Grades 11-12) – and teachers must initially qualify in two adjacent divisions. Thus, teacher education programs at university faculties of education must be organised in three categories: Primary-Junior (Grades 1-6); Junior-Intermediate (Grades 4-10); and Intermediate-Senior (Grades 7-12). This complex structure of teacher qualifications and teacher education programs was devised many decades ago and, while it is no longer relevant to either the curriculum or school organisation, there appears little appetite to change it. For the purposes of this report, therefore, we maintain our focus on the educational needs of teachers at elementary schools and secondary schools, while recognising that, in practice, these needs must be addressed in the context of current regulatory structures.

Rather, we need to consider the real needs of teachers at both elementary and secondary levels and then undertake reforms to both pre-service and in-service teacher education programs accordingly. Moreover, the expansion of pre-service teacher education programs from one year to two, currently being undertaken at Ontario faculties of education, provides a unique opportunity for making such reforms a reality. Readers should also note that, while preservice teacher education programs are operated by university faculties of education, their accreditation is the responsibility of the Ontario College of Teachers (OCT), and that the OCT operates in turn within a regulatory framework set by the Ministry of Education. Responsibility for making changes may therefore be shared among two or even three institutional levels.

The level of mathematics knowledge of elementary school teachers has often been suggested as a contributing factor to the decline in numeracy skills. Currently, most university faculties of education require no postsecondary mathematics courses for P/J or J/I programs (other than for those choosing mathematics as their "teachable subject" at the J/I level). Thus, applicants to a teacher education program for the early years could have Grade 11 mathematics as their terminal mathematics class in secondary school and, on graduation, may be asked to teach mathematics to students up to Grade 8. Although these teacher candidates will have been exposed to mathematical topics such as fractions and ratios, they may still have limited mathematical understanding of their complexity and lack confidence in teaching these topics. At the same time, a requirement that teacher candidates take additional university mathematics courses seems inappropriate, since such courses are not usually designed to develop students' basic numeracy.

We propose a two-pronged approach to resolving this dilemma. First, we would suggest that Ontario follow the lead of the Department for Education in England in instituting what are called "professional skills tests" designed "to ensure all teachers are competent in numeracy and literacy, regardless of their specialism."[52] Such an assessment for teacher candidates need not be used as a gatekeeper but rather as a means to identify individuals who should remediate their own skills prior to becoming licensed to teach. The second part of this approach is to encourage universities to offer mathematics courses of an appropriate type to provide elementary teachers with both the basic mathematical knowledge and skill and also a deeper understanding of the place of mathematics in real-life problem solving. Once such courses are generally available, faculties of education should increase their admission requirements accordingly.

However, there is no simple correlation between the number of post-secondary mathematics courses taken and the effectiveness of a mathematics teacher. While more instructional hours in mathematics and mathematics pedagogy is part of the solution, we also need to answer the question: what mathematics do teachers need to know in order to teach mathematics? The considerable body of research on this and the experience of colleagues in Quebec suggests that it is not simply subject content knowledge, but also the connection between that and the teaching of mathematics, which is critical.[55]

Beyond knowing how to do the mathematics, teachers need to have a deep understanding of a mathematics concept and its connections to other concepts, to know the why behind the mathematics enabling them to understand where their students might face challenges when exposed to the content.

[51] This was undertaken in 2013 and 2014 by the Minister of Education.

[52] http://sta.education.gov.uk/

This unpacking of a mathematics concept to examine and understand the landscape that surrounds it, is entirely different from the mathematics done by mathematicians, who focus on compressing or simplifying concepts. Teachers should engage in discussions about different ways that the content can be taught to support student understanding. While many teacher education programs centre on teaching behaviours, of greater importance is the learning behaviours of students so that teachers can become adept at creating an environment for learning through their teaching.

While in their teacher education programs, teacher candidates should be exposed to mathematics education research, both exploring it and conducting their own to help inform their practice. This early exposure to educational research may encourage them, once in the field to seek out research to keep their practice current with respect to mathematics concepts, methods of instruction, embedding technology, and assessment. It would also allow teacher candidates to connect with others in the field of mathematics and mathematics teaching.

To this point, we have focused on the needs of preservice teachers for elementary schools (P/J and J/I programs). Secondary school teachers, who are typically mathematics majors at the undergraduate level, have quite different needs. If the problem for elementary teachers is "not enough math" the problem for secondary teachers can sometimes be "too much math"! The mathematics taken as part of a bachelor's degree in mathematics is very far removed from the numeracy focus that we have proposed for secondary school mathematics courses in Grades 9 and 10 and for the majority of students in Grades 11 and 12 as well. This new focus requires not only an understanding of mathematics concepts but also a knowledge of real world professional and personal problem contexts in which mathematical concepts are embedded, together with the skill of unpacking those contexts to enable the embedded mathematical problems to emerge and be solved. For many highly numerate individuals, including many mathematics teachers, this knowledge and skill is so intuitive that they are able to undertake such problem solving without much thought and the struggles of others less numerate are sometimes hard to understand. The secondary school mathematics teacher requires more practice at identifying and solving problems from the real world (as opposed to the artificial ones often found in mathematics textbooks).

Finally, we must acknowledge that if teacher education for mathematics teaching is reformed only at the preservice level, then it will take a generation to impact the teaching of mathematics in the classroom. In-service teacher education is probably more important in the short run to make changes of the sort contemplated here. Yet this is also the most difficult to provide in ways that support without coercion and educate in ways that find real changes adopted in the classroom.

Traditional professional development events where teachers spend a day out of the classroom, then return to the classroom to implement a new strategy are the least effective[56]. Rather, professional development has been shown to improve teaching and learning when it is school-based, ongoing, and collaborative, and that which specifically focuses on the mathematics curriculum and ways to engage students with it. Support is needed for ongoing professional development which includes professional learning communities of teachers of mathematics and cross curricular communication to ensure that mathematics and numeracy applications are well understood and included in instruction in all subject areas.

In-service teacher education is obviously important. However, even this will only impact those who choose to take part in it and who change their practice as a result. Meanwhile, we should also note that there are already many teachers in the elementary schools of Ontario who are experienced, who have a record of teaching mathematics effectively, and who enjoy teaching it. The other side of the same coin is that there are also those, who – for whatever reason, whether or not they would admit it openly – would rather not have to teach mathematics at all. They love being teachers and they enjoy some parts of the curriculum, maybe language and the arts, but mathematics and the responsibility of teaching mathematics is something that weighs heavily on them. Even the most effective in-service teacher education programs are not going to turn around the math-phobias of many teachers. It is important that the schools address this reality both for the sake of those teachers and also, even more importantly, for the sake of the students in their classrooms who can pick up negative attitudes towards the subject, which in turn affect their achievement.

The most common arrangement for deploying staff in a school up to Grade 6 and in some schools in Grades 7 and 8 is for one teacher to be responsible for teaching the full curriculum (with the exception of certain subjects such as French). That means that students in classes where the teacher really enjoys teaching mathematics have the benefit of such a teacher's knowledge and skill, while those in other classes do not. We propose that elementary schools be encouraged to organise their classes at all grade levels (but particularly in Grades 4 through 8) in such a way that every class has (at least) two teachers assigned, one to teach mathematics, science and technology and the other to teach language and social studies. Thus, every teacher would have responsibility for two classes but for only half the curriculum – their preferred half – with each class.

Of course, some schools already have such an arrangement at least for some grade levels, but we believe that its general adoption would meet the needs of all students. Such a policy would not have any additional cost associated with it as the same number of teachers would teach the same number of classes as at present. But it would ensure that every student would be learning in the best possible mathematical teaching environment.

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[53] Ingrid Peretz. "Quebec might hold the formula to better nationwide math scores." *The Globe and Mail,* Dec 6 2013.

[54] Professor Annie Savard, email communication, December 29, 2013.

[55] For example: Deborah Ball, Heather Hill and Hyman Bass. "Knowing Mathematics for Teaching: Who knows mathematics well enough to teach third grade, and how can we decide?" *American Educator* (Fall 2005): 14-46. Heather Hill and Deborah Ball. "The curious – and crucial – case of mathematical knowledge for teaching." *Kappan* (October 2009): 68-71.

[56] As shown by such research as , for example: http://files.eric.ed.gov/fulltext/ED520732.pdf or http://files.eric.ed.gov/fulltext/EJ960950.pdf

Research

If the march towards a more numerate Ontario is to be achieved, then a wide variety of attitudes, policies and practices need to change as we have noted. More research is also required, particularly in three distinct areas:

- Monitoring numeracy levels among Ontario children and adults;
- Analysis of the uses of mathematics concepts and skills in practical fields of study and occupations;
- Research into effective classroom teaching of numeracy in Ontario and into effective ways to stimulate its spread within and among schools.

Monitoring numeracy levels is necessary to ensure that progress is being made on the broad goal of increasing numeracy for all. Not only do we need to do this in a general sense but also to enable us to identify schools where numeracy is increasing most strongly, in order to learn from their experience. EQAO tests are one measure but as we have noted numeracy is not just a matter of achievement of the mathematics curriculum but, especially in secondary school and beyond, it involves solving real-world practical problems where the mathematics is sometimes deeply embedded and contextual.

That thought leads to a second, quite different area of research. Many of the problems given to students in mathematics classes are not very authentic as we noted earlier in this paper and, in turn, not very engaging for students. Research is needed on fields of professional practice, on fields of study at postsecondary level, and on aspects of personal and family life to identify real situations in which mathematical concepts and skills are used to solve open-ended problems. These need to be "captured" and made available for use in the teaching of mathematics at the appropriate grade level. These resources can be provided through a web site to teachers throughout Ontario, along with suggestions for how best to use them in classrooms.

The third area of research that we consider to be important is focussed on methods of classroom teaching that promote positive attitudes towards mathematics as well as high achievement. Teachers need to have models of good and effective practice to build on and while some of this research is already being conducted, more is needed along with better dissemination of that which has been completed. Where our proposal to assign multiple teachers to classes is adopted, it would be very useful to gather data on the results of such an experiment, particularly in its pilot locations and years.

Next Steps

Closing the numeracy gap in Ontario is not going to be an easy task and not one for which the Ministry of Education can be held 100% responsible. The task requires the support and mandate from the Premier and Government of Ontario but it also requires the support and contributions of a wide variety of partners and stakeholders. We propose the immediate formation of a **provincial roundtable on numeracy** to develop an overall numeracy strategy for Ontario and to advise on its implementation.

The use of roundtables to address complex and multifaceted challenges that do not fit neatly into the responsibility of a single government department is a time-honoured tradition in Ontario. It permits the bringing together a variety of expertise from the private and voluntary sectors as well as from the public sector. It enables wide-ranging discussions, specially commissioned research and the development of creative solutions. Closing the numeracy gap is exactly such a challenge and we call on the Premier of Ontario to create such a roundtable.

APPENDIX A


THE ESSENTIALS OF NUMERACY
National Numeracy
for everyone, for life
HANDLING INFORMATION
REASONING
PROBLEM SOLVING
NUMBERS (AND THE NUMBER SYSTEM)
BEING NUMERATE
SHAPE, SPACE AND MEASURES
DECISION MAKING
OPERATIONS AND CALCULATIONS